Integration- Mark Scheme

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)}+\frac{1}{(k)}=\left(-\frac{1}{3}+1\right)\times\frac{1}{k}\propto\frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to *k*.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \rightarrow \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

2.

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H} dH = \int \frac{\cos 0.25t}{40} dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t = 0, H = 5 \Rightarrow c = \ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int \frac{1}{H} dH = \int \frac{\cos 0.25t}{40} dt$ or equivalent.

The integral signs need to be present on both sides and the dH AND dt need to be in the correct positions.

M1: Integrates both sides to reach $\ln H = A\sin 0.25t$ or equivalent with or without the $+ c$

A1: $\ln H = \frac{1}{10}\sin 0.25t + c$ or equivalent with or without the $+ c$. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H = 4\sin 0.25t + c$ or equivalent.

dM1: Substitutes $t=0, H=5 \Rightarrow c=..$ There needs to have been a single "$+c$" to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H = 4\sin 0.25t + c \Rightarrow\Rightarrow H^{40} = e^{4\sin 0.25t} + e^c \Rightarrow 5^{40} = 1 + e^c \Rightarrow c = ...$

Also many students will be attempting to get to the given answer so condone the method of finding c =... These students will lose the A1* mark

A1*: Proceeds via $\ln H = \frac{1}{10}\sin 0.25t + \ln 5$ or equivalent to the given answer $H = 5e^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone c's going to c's when they should be e^c or A

Accept as a minimum $\ln H = \frac{1}{10}\sin 0.25t + \ln 5 \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + \ln 5}$ or $H = e^{\frac{1}{10}\sin 0.25t} \times e^{+\ln 5}$ before sight of the given answer

If the only error was to omit the integration signs on line 1, thus losing the first M1, allow the candidate to have access to this mark following a correct intermediate line (see above).

If they attempt to change the subject first then the constant of integration must have been adapted if the A1* is to be awarded. $\ln H = \frac{1}{10}\sin 0.25t + c \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + c} \Rightarrow H = Ae^{\frac{1}{10}\sin 0.25t}$

The dM1 and A1* under this method are awarded at virtually the same time.

Also, for the final two marks, you may see a proof from $\int_0^H \frac{40}{H}\,dH = \int_5^t \cos 0.25t\,dt$

There is an alternative via the use of an integrating factor.

(b)

B1: States that the maximum height is 5.53 m Accept $5e^{0.1}$ Condone a lack of units here, but penalise if incorrect units are used.

(c)

M1: For identifying that it would reach the maximum height for the 2nd time when $0.25t = \frac{5\pi}{2}$ or 450

A1: Accept awrt 31.4 or 10π Allow if units are seen

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3.

Question Number	Scheme	Marks
3.	x: 0, 0.5, 1, 1.5, 2 y: 1, 2.821, **6**, 12.502, 26.585	
(a)	$\{\text{At } x=1,\}\ y = 6$ (allow 6.000 or even 6.00)	B1 cao
		(1)
(b)	$\frac{1}{2}\times 0.5$;	B1 oe
	$\underline{\{1 + 26.585 + 2(2.821 + \text{their } 6 + 12.502)\}}$ For structure of $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1<u>A1ft</u>
	$\frac{1}{2}\times 0.5\ \underline{\{1 + 26.585 + 2(2.821 + 6 + 12.502)\}}\ \{= \frac{1}{4}(70.231) = 17.557..\} =$ awrt 17.56	A1
		(4)
(c)	10 + "17.56" = "27.56"	B1ft
		(1)
		[6]

	Notes
(a)	B1: 6
(b)	B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent.
	M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values
	A1ft: for the correct bracket $\{\ldots\ldots\}$ following through candidate's y value found in part (a).
	A1: for answer which rounds to 17.56
	NB: Separate trapezia may be used: B1 for 0.25, M1 for 1/2 $h(a + b)$ used 3 or 4 times (and A1ft if it is all correct) Then A1 as before.
	Special case: Bracketing mistake $0.25\times(1 + 26.585) + 2(2.821 + \text{their } 6 + 12.502)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 49.542 usually indicates this error.
(c)	B1ft: 10 + their answer to part (b) (May be obtained by using the trapezium rule again with all values for y increased by 5)

4.

Question Number	Scheme	Marks
10. (a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
	Either: Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$ Or: Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$	A1
	Either: So turning point (all correct work so far) Or: Deduce $x = 1$ from correct work	A1cso (3)
(b) Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle ABP = $\frac{1}{2} \times 1 \times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x\ \{+c\}$ *or* $x^4 + 3x^3 - 15x^2 - 8x\ \{+c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{1} = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ $= (-19) - \frac{261}{256}$ or $-19 - 1.02$	dM1
	So Area = "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + "20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	= 32.52 (NOT − 32.52)	A1 (7) **[10]**
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2 For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the −25	B1
	For second mark: Integrating to find triangle area $\int_1^2 (25x - 50)dx = \left[\frac{25}{2}x^2 - 50x\right]_1^2 = -50 + 37.5 = -12.5$ so area is 12.5	B1
	Then mark as before if they use Method in original scheme	
(b) Way 2	**Way 2:** Those who use area for **original curve between -1/4 and 2** and subtract area **between line and curve between 1 and 2** have a correct (long) method .	
	The first B1 (if y=−25 is not seen) is for equation of straight line $y = 25x - 50$	B1
	The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5	B1
	$\int (4x^3 + 9x^2 - 55x + 42)dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\ \{+c\}$ (or integration as in Way 1)	M1A1
	The dM1 is for correct use of the different correct limits for each of the two areas: i.e. $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2} = (16 + 24 - 60 - 16) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ **And** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2 = 16 + 24 - 110 + 84 - (1 + 3 - 27.5 + 42)$	dM1
	So Area =their $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2}$ **minus their** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2$ i.e. "*their* 37.0195" − "*their* 4.5" (with both sets of limits correct for the integral)	ddM1
	Reaching = 32.52 (NOT − 32.52)	A1
	See over for special case with wrong limits	

	NB: Those who attempt curve – line wrongly with limits −1/4 to 2 may earn M1A1 for correct integration of their cubic. Usually e.g. $\int (4x^3+9x^2-55x+42)\,dx = x^4+\frac{9}{3}x^3-\frac{55x^2}{2}+42x\,\{+c\}$ (They will not earn any of the last 3 marks) They may also get first B1 mark for the correct equation of the straight line (usually seen but may be implied by correct line −curve equation) and second B1 if they also use limits 1 and 2 to obtain 4.5 (or find the triangle area 12.5).	M1A1

	Notes
(a)	M1: Attempt at differentiation - all powers reduced by 1 with $8 \to 0$. A1: the derivative must be correct and uses derivative = 0 to find x or substitutes x = 1 to give 0. Ignore any reference to the other root (−5/2) for this mark. A1cso: obtains x = 1 from correct work, or deduces turning point (if substitution used – may be implied by a preamble e.g. dy/dx =0 at T.P.) **N.B**. If their factorisation or their second root is incorrect then award A0cso. If however their factorisation/roots are correct, it is not necessary for them to comment that −2.5 is outside the range given.
(b)	**Way 1:** B1: Obtains y = - 25 when x = 1 (may be seen anywhere – even in (a)) or finds correct equation of line is y = 25x - 50 B1: Obtains area of triangle = 12.5 (may be seen anywhere). Allow −12.5. Accept $\frac{1}{2}\times 1\times 25$ M1: Attempt at integration of cubic; two correct terms for their integration. No limits needed A1: completely correct integral for the cubic **(may be unsimplified)** dM1: We are looking for the start of a correct method here (dependent on previous M). It is for substituting 1 and -1/4 and subtracting. May use 2 and -1/4 and also 2 and 1 **AND subtract** (which is equivalent)
	ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so **adds** two positive numbers (areas) together – one is area of triangle, the other is area of region obtained from integration of correct function with correct limits (may add two negatives then makes positive) **Way 2:** This is a long method and needs to be a correct method B1: Finds y=−25 at x=1 ,or correct equation of line is $y = 25x - 50$ B1: May be implied where WAY 2 is used and final correct answer obtained so award of final A1 results in the award of this B1. It may also be implied by correct integration of line equation or of curve minus line expression between limits 1 and 2. So if only slip is final subtraction (giving final A0, this mark may still be awarded) So may be implied by 4.5 seen for area of "segment shaped" region between line and curve. M1: Attempt at integration of given cubic or after attempt at subtracting their line equation (no limits needed). Two correct terms needed A1: Completely correct integral for their cubic **(may be unsimplified) – may have wrong coefficients of *x* and wrong constant term through errors in subtraction** dM1: **Use limits for original curve between -1/4 and 2 and use limits of 1 and 2 for area between line and curve–** needs completely correct limits– see scheme- this is dependent on two integrations ddM1: (depends on both method marks) Subtracts "*their* 37.0195" − "*their* 4.5" Needs consistency of signs. A1: 32.52 or awrt 32.52 e.g. $32\frac{133}{256}$ **NB: This correct answer implies the second B mark** (Trapezium rule **gets no marks after first two B marks) The first two B marks may be given wherever seen. The integration of a cubic gives the following M1 and correct integration of their cubic** $\int (4x^3+9x^2+Ax+B)\,dx = x^4+\frac{9}{3}x^3+\frac{Ax^2}{2}+Bx\,\{+c\}$ gives the A1

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5.

Question Number	Scheme		Marks
	$y = 8 - 2^{x-1},\ 0 \leq x \leq 4$		
2. (a)	7	7	B1 **cao**
			[1]
(b)	$\left(\int_0^4 (8 - 2^{x-1})\,dx \approx\right) \frac{1}{2} \times 1; \times \underline{\{7.5 + 2(\text{"their 7"} + 6 + 4) + 0\}}$	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$	B1;
		For structure of trapezium rule $\{\ldots\ldots\ldots\}$ for a candidate's y-ordinates.	M1
	$\left\{= \frac{1}{2} \times 41.5\right\} = 20.75$ o.e.	20.75	A1 **cao**
			[3]
(c)	Area $(R) = \text{"20.75"} - \frac{1}{2}(7.5)(4)$		M1
	$= 5.75$	5.75	A1 **cao**
			[2]
			6

	Question 2 Notes	
(a)	**B1**	For 7 only
(b)	**B1**	For using $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent.
	M1	Requires the correct $\{\ldots\}$ bracket structure. It needs the 7.5 stated but the 0 may be omitted. The inner bracket needs to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however (unless it is 0)). M0 is awarded if values used in brackets are x values instead of y values
	A1	For 20.75 or fraction equivalent e.g. $20\frac{3}{4}$ *or* $\frac{83}{4}$
	Note	**NB: Separate trapezia may be used** : B1 for 0.5, M1 for 1/2 $h(a + b)$ used 3 or 4 times Then A1 as before.
	Special case:	Bracketing mistake $0.5 \times (7.5 + 0) + 2($ their $7 + 6 + 4)$ scores B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 37.75 usually indicates this error.
	Common error:	Many candidates use $\frac{1}{2} \times \frac{4}{5}$ and score B0 Then they proceed with $\underline{\{7.5 + 2(\text{"their 7"} + 6 + 4) + 0\}}$ and score M1 This usually gives 16.6 for B0M1A0
(c)	**M1**	their answer to (b) $-$ area of triangle with base 4 and height 7.5 or alternative correct method e.g. their answer to (b) $- \int_0^4 \left(7.5 - \frac{7.5}{4}x\right) dx$ (Even if this leads to a negative answer) This may be implied by a correct answer or by an answer where they have subtracted 15 from their answer to part (b). Must use answer to part (b).
	A1	5.75 or fraction equivalent e.g. $5\frac{3}{4}$ *or* $\frac{23}{4}$

6.

Question Number	Scheme		Marks
7. (a)	$\left\{\int\left(3x - x^{\frac{3}{2}}\right)dx\right\} = \frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\{+c\}$	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$	M1
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
			[3]
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$	Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	M1
	$\left\{\text{Area}(S) = \left[\frac{3x^2}{2} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$		
	$= \left(\frac{3(9)^2}{2} - \left(\frac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$	Applies the limit 9 on an integrated function with **no wrong lower limit**.	ddM1
	$\left\{= \left(\frac{243}{2} - \frac{486}{5}\right) - \{0\}\right\} = \frac{243}{10}$ or 24.3	$\frac{243}{10}$ or 24.3	A1 oe
			[3]
			6

	Question 7 Notes	
(a)	**M1**	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$
	1st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.
	2nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the n+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+ c$". Ignore integral signs in their answer.
(b)	**1st M1**	Sets $y = 0$, and reaches the **correct** $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$) Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$. Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit **9** substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.
	A1	$\frac{243}{10}$ or 24.3
	Common Error	**Common Error** $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ *so* $x = \sqrt{3}$ **Then** uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3

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7.

Question Number	Scheme	Marks
	May mark (a) and (b) together	
6. (a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ $(+c)$	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+c)$	A1cao (4)
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either ± -32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\lvert\int_0^4 ydx\right\rvert + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 (5) [9]

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need (+ *c*)

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need (+ *c*)

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

d**M1:** (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A\times 9^{\frac{5}{2}} - B\times 9^2$ with $A\times 4^{\frac{5}{2}} - B\times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4\times 4^{\frac{5}{2}} - 10\times 4^2 + 4\times 9^{\frac{5}{2}} - 10\times 9^2 - 4\times 4^{\frac{5}{2}} - 10\times 4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5

This also applies if 4 never seen.

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8.

Question Number	Scheme			Marks
1.(a)	x: 1, 1.25, 1.5, 1.75, 2 y: 1.414, **1.601**, 1.803, 2.016, 2.236			
	$\{\text{At } x = 1.25,\}\ y = 1.601$ (only)		1.601 **(May not be in the table and can score if seen as part of their working in (b))**	B1 cao
				[1]
(b)	$\frac{1}{2}\times 0.25;\times\left\{\underline{1.414+2.236+2(\text{their } 1.601+1.803+2.016)}\right\}$			B1; M1 A1ft
	B1; for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent.	M1: Structure of $\{\ldots\ldots\ldots\ldots\}$	A1ft: for the correct expression as shown following through candidate's y value found in part (a).	
	M1 requires the correct structure for the y values. It needs to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(…..) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are x values instead of y values. A1ft: for the correct underlined expression as shown following through candidate's y value found in part (a). Bracketing mistakes: e.g. $\left(\frac{1}{2}\times\frac{1}{4}\right)(1.414+2.236)+2(\text{their } 1.601+1.803+2.016)(=11.29625)$ $\left(\frac{1}{2}\times\frac{1}{4}\right)1.414+2.236+2(\text{their } 1.601+1.803+2.016)(=13.25275)$ Both score B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks could be given). **Alternative:** Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{8}(1.414+1.601)+\frac{1}{8}(1.601+1.803)+\frac{1}{8}(1.803+2.016)+\frac{1}{8}(2.016+2.236)\right]$ B1 for $\frac{1}{8}$ (aef), M1 for correct structure, 1st A1ft for correct expression, ft their 1.601			
	$\{=\frac{1}{8}(14.49)\}=1.81125$		1.81 or awrt 1.81	A1
	Correct answer only in (b) scores no marks **If required accuracy is not seen in (a), full marks can still be scored in (b) (e.g. uses 1.6)**			
				[4]
				Total 5

9.

Question Number	Scheme		Marks
4.	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$	M1: $x^n \to x^{n+1}$ A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$. A1: $\frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ or equivalent. e.g. $\frac{\frac{x^4}{6}}{4}+\frac{\frac{x^{-1}}{3}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)	M1A1A1
	Note that some candidates may change the function prior to integrating e.g. $\int\frac{x^3}{6}+\frac{1}{3x^2}\mathrm{d}x=\int 3x^5+6\mathrm{d}x$ **in which case allow the M1 if** $x^n \to x^{n+1}$ **for their changed function and allow the M1 for limits if scored**		
	$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(\sqrt{3})^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$		**dM1**
	2nd **dM1**: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. **The 2nd M1 is dependent on the 1st M1 being awarded.**		
	$=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$	$\frac{2}{3}-\frac{1}{9}\sqrt{3}$ or $a=\frac{2}{3}$ and $b=-\frac{1}{9}$. Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do **not** allow $\frac{6-\sqrt{3}}{9}$	A1**cso**
	This final mark is cao and cso – there must have been no previous errors		
			Total 5
	Common Errors (Usually 3 out of 5)		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \int\left(\frac{x^3}{6}+3x^{-2}\right)\mathrm{d}x = \frac{x^4}{6(4)}+\frac{3x^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{3(\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{3(1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{3}{\sqrt{3}}\right)-\left(\frac{1}{24}+\frac{3}{-1}\right)=\frac{10}{3}-\sqrt{3}$ A0		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \int\left(\frac{x^3}{6}+(3x)^{-2}\right)\mathrm{d}x = \frac{x^4}{6(4)}+\frac{(3x)^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(3\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{(3\times 1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{\sqrt{3}}{9}$ A0 **Note this is the correct answer but follows incorrect work.**		

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10.

Question Number	Scheme	Marks
4.	x: 0, 0.5, 1, 1.5, 2, 2.5, 3 ; y: 5, 4, 2.5, 1.538, 1, 0.690, 0.5	
(a)	$\{\text{At } x = 1.5,\}\ y = 1.538$ (only)	B1 **cao**
		[1]
(b)	$\frac{1}{2} \times 0.5$;	B1 oe
	$\{5 + 0.5 + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}$ For structure of $\{\ldots\ldots\ldots\}$;	M1A1ft
	$\frac{1}{2} \times 0.5 \times \{(5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)\}$ $\{= \frac{1}{4}(24.956) = 6.239\}$ = awrt 6.24	A1
		[4]
(c)	Adds Area of Rectangle or first integral $= 3 \times 4$ or $[4x]_0^3$ to **previous answer**	M1
	So required estimate = $\{"6.239" + 12 = "18.239"\}$ = "awrt 18.24" (or 12 + previous answer).	A1ft
	N.B. 7 × 4 + previous answer is M0A0 (added 4 seven times because 7 numbers in table)	**[2]**
		7

	Notes for Question 4
(a)	B1: 1.538
(b)	B1: for using $\frac{1}{2} \times 0.5$ or $\frac{1}{4}$ or equivalent. M1: requires the correct $\{\ldots\}$ bracket structure. It needs the first bracket to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values A1ft: for the correct bracket $\{\ldots\}$ following through candidate's y value found in part (a). A1: for answer which rounds to 6.24. NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 $h(a + b)$ used 5 or 6 times (and A1ft if it is all correct) Then A1 as before. Special case: Bracketing mistake $0.25 \times (5 + 0.5) + 2(4 + 2.5 + \text{their } 1.538 + 1 + 0.690)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 20.831 usually indicates this error.
(c)	M1: Relates **previous answer (not integral of previous answer**) to this question by integrating 4 between limits, and adding, or by using geometry to find rectangle and adding. A1ft: for 12 + answer to (b)
Alternative method (c)	Those who do a trapezium rule for part (b)- using the table from (a) with 4 added to each cell of the table Get: M1 for "*their* $\frac{1}{4}$"$\times\{9 + 4.5 + 2(8 + 6.5 + \text{their } 5.538 + 5 + 4.690)\}$ = (structure must be correct – allow one copying error only) And A1ft: for awrt 18.24 (or 12 + previous answer).

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11.

Question Number	Scheme		Marks
9.	$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$		
(a)	6.272 , 3.634	Awrt in each case	B1, B1
	Special case 6.27 and 3.63 scores B1B0		
			(2)
(b)	$\frac{1}{2}\times\frac{1}{2}$ or $\frac{1}{4}$		B1
	$....\{(0+0)+2(5.866+"6.272"+5.210+"3.634"+1.856)\}$	Need {} or implied later for A1ft	M1A1ft
	(0 + 0) may be implied if omitted and follow through their f(2) and f(3) in an otherwise correct expression and allow one missing or mis-copied term in the 2(....) bracket for the **method** mark		
	$\frac{1}{2}\times 0.5(0+0)+2(5.866+"6.272"+5.210+"3.634"+1.856)$ **Unless followed by an answer that implies correct (missing) brackets, scores B1M1A0A0 (Usually implied by an answer of 45.676)**		
	$\frac{1}{2}\times 0.5\{(0+0)+2(5.866+"6.272"+5.210+"3.634"+1.856)\}$ $= \frac{1}{4}\times 45.676$		
	= 11.42	cao	A1
	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct)		
	NB $\frac{1}{2}\times 0.5\{(0+0)+2(0+5.866+"6.272"+5.210+"3.634"+1.856+0)\}$ **Scores B1M0A0A0**		
	Correct answer with no working scores 0/4		
			(4)
(c)	$\int y\,dx = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1}(+c)$	M1: $x^n \to x^{n+1}$ on any term A1: $27x - x^2$ A1: $-6x^{\frac{3}{2}}$ A1: $+16x^{-1}$	M1A1A1A1
	Accept any correct and possibly unsimplified versions for the terms and mark **in this order** on Epen		
	$\left(27(4)-(4)^2-6(4)^{\frac{3}{2}}+16(4)^{-1}\right)$ $-\left(27(1)-(1)^2-6(1)^{\frac{3}{2}}+16(1)^{-1}\right)$	**Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1. May be implied by 48 – 36 but you may need to check both their values if the integration has errors.**	dM1
	= (48 – 36)		
	12	Cao (Penalise -12)	A1
			(6)
			[12]

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

12.

Question number	Scheme	Marks
7 (a)	x: 0, 0.25, 0.5, 0.75, 1; y: 1, 1.251, **1.494**, **1.741**, 2	B1, B1 (2)
(b)	$\frac{1}{2}\times 0.25,\ \{(1+2)+2(1.251+1.494+1.741)\}$ o.e.	B1, M1,A1 ft
	$=1.4965$	A1 (4)
		6 marks

x	0	0.25	0.5	0.75	1
y	1	1.251	**1.494**	**1.741**	2

Notes

(a) first **B1** for 1.494 and second **B1** for 1.741 (1.740 is **B0**)
Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b) **B1:** Need ½ of 0.25 or 0.125 o.e.
M1: requires first bracket to contain first plus last values **and** second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and **M** mark can be allowed (An extra repeated term forfeits the **M** mark however)
x values: **M0** if values used in brackets are x values instead of y values

A1ft follows their answers to part (a) and is for {correct expression}
Final **A1**: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table)
Separate trapezia may be used : **B1** for 0.125, **M1** for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and **A1**ft if it is all correct)
e.g.. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is **M1 A0**
equivalent to missing one term in { } in main scheme

Special Case: Bracketing mistake: i.e. 0.125(1+2) +2(1.251+1.494+1.741)
scores **B1 M1 A0 A0 for 9.347** If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given).
Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review

Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0
B1M1A1ft then A1 (lose 1 mark)

NB Bracket is 11.972

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13.

Question number	Scheme	Marks
6: (a)	<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>y</td><td>16.5</td><td>7.361</td><td>**4**</td><td>**2.31**</td><td>1.278</td><td>0.556</td><td>0</td></tr></table>	B1, B1 (2)
(b)	$\frac{1}{2}\times 0.5,\ \{(16.5+0)+2(7.361+4+2.31+1.278+0.556)\}$	B1, M1A1ft
	= 11.88 (or answers listed below in note)	A1 (4)
(c)	$\int_1^4 \frac{16}{x^2}-\frac{x}{2}+1\,dx = \left[-\frac{16}{x}-\frac{x^2}{4}+x\right]_1^4$	M1 A1 A1
	$=[-4-4+4]-[-16-\frac{1}{4}+1]$	dM1
	$=11\frac{1}{4}$ or equivalent	A1 (5) **11**

Notes	(a) **B1** for 4 or any correct equivalent e.g. 4.000 **B1** for 2.31 or 2.310 (b) **B1:** Need 0.25 or ½ of 0.5 **M1:** requires first bracket to contain first y value plus last y value (0 may be omitted or be at end) **and** second bracket to include no additional y values from those in the scheme. They may however omit one value as a slip. N.B. Special Case - Bracketing mistake $\frac{1}{2}\times 0.5(16.5+0)+2(7.361+4+2.31+1.278+0.556)$ scores **B1 M1 A0 A0** unless the final answer implies that the calculation has been done correctly (then full marks) A1ft: This should be correct but ft their 4 and 2.31 **A1**: Accept 11.8775 or 11.878 or 11.88 only (c) **M1** Attempt to integrate ie power increased by 1 or 1 becomes x , **A1** two correct terms, next **A1** all three correct unsimplified (ignore +c) (Allow $-16x^{-1}-0.25x^2+1x$ or equivalent) **dM1 (This cannot be earned if previous M mark has not been awarded)** Uses limits 4 and 1 in their integrated expression and subtracts (either way round) **A1** 11.25 or 11 ¼ or 45/4 or equivalent (penalise negative final answer here)
Alternative Method for (b)	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct for their "4" and "2.31") final A1 for 11.88 etc. as before
	In part (b) Need to use trapezium rule – answer only (with no working) is 0/4 -any doubts send to review In part (c) need to see integration

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

14.

Question Number	Scheme	Marks
6. **(a)**	$\begin{array}{c\|c\|c\|c\|c\|c} x & 2 & 2.25 & 2.5 & 2.75 & 3 \\ \hline y & 0.5 & 0.38 & 0.298507\ldots & 0.241691\ldots & 0.2 \end{array}$	
	At $\{x = 2.5,\}$ $y = 0.30$ (only) — At least one y-ordinate correct.	B1
	At $\{x = 2.75,\}$ $y = 0.24$ (only) — Both y-ordinates correct.	B1
		(2)
(b)	Outside brackets $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$	B1 aef
	$\frac{1}{2}\times 0.25\ ;\times\underline{\{0.5+0.2+2(0.38+\text{ their } 0.30+\text{ their } 0.24)\}}$ — <u>For structure of</u> $\{\ldots\ldots\ldots\ldots\}$;	M1
	Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant".	<u>A1</u> $\sqrt{}$
	$\{= \frac{1}{8}(2.54)\} = \text{awrt } 0.32$ — awrt 0.32	A1
		(4)
(c)	Area of triangle $= \frac{1}{2}\times 1\times 0.2 = 0.1$	B1
	$\text{Area}(S) = \text{"}0.3175\text{"} - 0.1$	M1
	$= 0.2175$	A1 ft
		(3)
		[9]

Question Number	Scheme	Marks
	<u>**Notes**</u>	
(b)	B1 for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent. M1 requires the correct $\{\ldots\ldots\}$ bracket structure. This is for the first bracket to contain first y-ordinate plus last y-ordinate and the second bracket to be the summation of the remaining y-ordinates in the table. No errors (eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y-ordinate) are allowed in the second bracket and the second bracket must be multiplied by 2. Only one copying error is allowed here in the $2(0.38+\text{ their } 0.30+\text{ their } 0.24)$ bracket. A1ft for the correct bracket $\{\ldots\ldots\}$ following through candidate's y-ordinates found in part (a). A1 for answer of awrt 0.32 . <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** then award M1A0A0 for either $\frac{1}{2}\times 0.25\times 0.5+2(0.38+\text{ their } 0.30+\text{ their } 0.24)+0.2$ (nb: yielding final answer of 2.1025) so that the 0.5 is only multiplied by $\frac{1}{2}\times 0.25$ or $\frac{1}{2}\times 0.25\times(0.5+0.2)+2(0.38+\text{ their } 0.30+\text{ their } 0.24)$	

	(nb: yielding final answer of 2.1025) so that the 0.5 is only multiplied by $\frac{1}{2}\times 0.25$ or $\frac{1}{2}\times 0.25\times(0.5+0.2)+2(0.38+\text{ their } 0.30+\text{ their } 0.24)$ (nb: yielding final answer of 1.9275) so that the $(0.5+0.2)$ is multiplied by $\frac{1}{2}\times 0.25$. **Need to see trapezium rule – answer only (with no working) gains no marks.** **<u>Alternative:</u>** Separate trapezia may be used, and this can be marked equivalently. (See appendix.)
(c)	B1 for the area of the triangle identified as either $\frac{1}{2}\times 1\times 0.2$ or 0.1. May be identified on the diagram. M1 for "part (b) answer" – "0.1 only" or "part (b) answer – their attempt at 0.1 only". (Strict attempt!) A1ft for correctly following through "part (b) answer" – 0.1. This is also dependent on the answer to (b) being greater than 0.1. Note: candidates may round answers here, so allow A1ft if they round their answer correct to 2 dp.

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

15.

Question Number	Scheme	Marks
1.	(a) 2.35, 3.13, 4.01 (One or two correct B1 B0, all correct B1 B1) <u>Important</u>: If part (a) is blank, or if answers have been crossed out and no replacement answers are visible, please send to Review as 'out of clip'.	B1 B1 (2)
	(b) $\frac{1}{2}\times 0.2$ ...... (or equivalent numerical value) $k\{(1+5)+2(1.65+p+q+r)\}$, k constant, $k\neq 0$ (See notes below) $=2.828$ (awrt 2.83, allowed even after minor slips in values) The fractional answer $\frac{707}{250}$ (or other fraction wrt 2.83) is also acceptable. Answers with no working score no marks.	B1 M1 A1 A1 (4) **6**

	(a) Answers must be given to 2 decimal places. No marks for answers given to only 1 decimal place. (b) The p, q and r below are positive numbers, none of which is equal to any of: 1, 5, 1.65, 0.2, 0.4, 0.6 or 0.8 M1 A1: $k\{(1+5)+2(1.65+p+q+r)\}$ M1 A0: $k\{(1+5)+2(1.65+p+q)\}$ or $k\{(1+5)+2(p+q+r)\}$ M0 A0: $k\{(1+5)+2(1.65+p+q+r+other\,value(s))\}$ Note that if the only mistake is to omit a value from the second bracket, this is considered as a slip and the M mark is allowed. Bracketing mistake: i.e. $\frac{1}{2}\times 0.2(1+5)+2(1.65+2.35+3.13+4.01)$ instead of $\frac{1}{2}\times 0.2\{(1+5)+2(1.65+2.35+3.13+4.01)\}$, so that only the (1 + 5) is multiplied by 0.1 scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). Alternative: Separate trapezia may be used, and this can be marked equivalently.	

16.

Question Number	Scheme	Notes	Marks
3.	x: 0, 0.2, 0.4, 0.6, 0.8, 1 ; y: 2, ***1.8625426...***, 1.71830, 1.56981, 1.41994, 1.27165	$y = \dfrac{6}{(2+e^x)}$	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	1.86254	B1 cao
	Note: Look for this value on the given table or in their working.		**[1]**
(b)	$\frac{1}{2}(0.2)\left[2+1.27165+2\left(\text{their } 1.86254+1.71830+1.56981+1.41994\right)\right]$	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$ or $\frac{1}{2}\times\frac{1}{5}$	B1 o.e.
		For structure of $[\,.......\,]$	M1
	$\left\{=\frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	anything that rounds to 1.6413	A1
			[3]
(c)	$\{u = e^x \text{ or } x = \ln u \Rightarrow\}$		
	$\frac{du}{dx} = e^x$ **or** $\frac{du}{dx} = u$ **or** $\frac{dx}{du} = \frac{1}{u}$ **or** $du = u\,dx$ etc., **and** $\int \frac{6}{(e^x+2)}dx = \int \frac{6}{(u+2)u}du$	**See notes**	B1 *
	$\{x=0\} \Rightarrow a = e^0 \Rightarrow \underline{a=1}$ $\{x=1\} \Rightarrow b = e^1 \Rightarrow \underline{b=e}$	$a=1$ and $b=e$ or $b=e^1$ or evidence of $0\to 1$ and $1\to e$	B1
	NOTE: 1st B1 mark CANNOT be recovered for work in part (d) **NOTE: 2nd B1 mark CAN be recovered for work in part (d)**		**[2]**
(d) **Way 1**	$\frac{6}{u(u+2)} \equiv \frac{A}{u}+\frac{B}{(u+2)}$ $\Rightarrow 6 \equiv A(u+2)+Bu$	Writing $\frac{6}{u(u+2)} \equiv \frac{A}{u}+\frac{B}{(u+2)}$, o.e. **or** $\frac{1}{u(u+2)} \equiv \frac{P}{u}+\frac{Q}{(u+2)}$, o.e., and a complete method for finding the value of at least one of **their** A or **their** B (or **their** P or **their** Q)	M1
	$u=0 \Rightarrow A=3$ $u=-2 \Rightarrow B=-3$	Both **their** $A=3$ **and their** $B=-3$. (Or **their** $P=\frac{1}{2}$ **and their** $Q=-\frac{1}{2}$ with the factor of 6 in front of the integral sign)	A1
	$\int \frac{6}{u(u+2)}du = \int\left(\frac{3}{u}-\frac{3}{(u+2)}\right)du$	Integrates $\frac{M}{u}\pm\frac{N}{u\pm k}$, $M, N, k \neq 0$; (i.e. ***a two term partial fraction***) to obtain either $\pm\lambda\ln(\alpha u)$ **or** $\pm\mu\ln(\beta(u\pm k))$; $\lambda, \mu, \alpha, \beta \neq 0$	M1
	$= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	Integration of both terms is **correctly followed through** from **their** M and from **their** N.	A1 ft
	$\left\{\text{So}\left[3\ln u - 3\ln(u+2)\right]_1^e\right\}$ $= \left(3\ln(e) - 3\ln(e+2)\right) - \left(3\ln 1 - 3\ln 3\right)$ [**Note:** A proper consideration of the limit of $u=1$ is required for this mark]	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq 1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3-3\ln(e+2)+3\ln 3$ **or** $3(1-\ln(e+2)+\ln 3)$ **or** $3+3\ln\left(\frac{3}{e+2}\right)$ **or** $3\ln\left(\frac{e}{e+2}\right)-3\ln\left(\frac{1}{3}\right)$ **or** $3-3\ln\left(\frac{e+2}{3}\right)$ **or** $3\ln\left(\frac{3e}{e+2}\right)$ **or** $\ln\left(\frac{27e^3}{(e+2)^3}\right)$	**see notes**	A1 cso
	Note: Allow e^1 in place of e for the final A1 mark.		**[6]**
	Note: Give final A0 for $3-3\ln e+2+3\ln 3$ (i.e. bracketing error) unless recovered.		**12**
	Note: Give final A0 for $3-3\ln(e+2)+3\ln 3-3\ln 1$, where $3\ln 1$ has not been simplified to 0		
	Note: Give final A0 for $3\ln e-3\ln(e+2)+3\ln 3$, where $3\ln e$ has not been simplified to 3		

		Question 3 Notes
3. (b)	**Note**	**M1:** Do not allow an extra y-value ***or*** a repeated y value in their […] Do not allow an omission of a y-ordinate in their […] for M1 **unless** they give the correct answer of awrt 1.6413, in which case both M1 and A1 can be scored.
	Note	**A1:** Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.64150274…)
	Note	Full marks can be gained in part (b) for awrt 1.6413 even if B0 is given in part (a)
	Note	Award B1M1A1 for $\frac{1}{10}(2+1.27165)+\frac{1}{5}(\text{their } 1.86254+1.71830+1.56981+1.41994) = \text{awrt } 1.6413$
		<u>Bracketing mistakes:</u> Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2)+2+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165$ $(=16.51283)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2+1.27165)+2(\text{their } 1.86254+1.71830+1.56981+1.41994)$ $(=13.468345)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2)+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165$ $(=14.61283)$
		<u>Alternative method: Adding individual trapezia</u> $\text{Area} \approx 0.2\times\left[\frac{2+\text{"}1.86254\text{"}}{2}+\frac{\text{"}1.86254\text{"}+1.71830}{2}+\frac{1.71830+1.56981}{2}+\frac{1.56981+1.41994}{2}+\frac{1.41994+1.27165}{2}\right]$ $= 1.641283$ **B1** 0.2 and a divisor of 2 on all terms inside brackets **M1** First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 **A1** anything that rounds to 1.6413
3. (c)	**1st B1**	Must start from either • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ **and** state either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$ **and** end at $\int \frac{6}{u(u+2)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working.**
	Note	So, just writing $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **and** $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x=\int \frac{6}{u(u+2)}\,\mathrm{d}u$ is sufficient for 1st B1
	Note	Give 2nd B0 for $b=2.718...$, without reference to $a=1$ and $b=\mathrm{e}$ or $b=\mathrm{e}^1$
	Note	You can also give the 1st B1 mark for using a reverse process. i.e. Proceeding from $\int \frac{6}{u(u+2)}\,\mathrm{d}u$ to $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, **with no incorrect working,** and stating either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$
3. (d)	**Note**	Give final A0 for $3-3\ln(\mathrm{e}+2)+3\ln 3$ simplifying to $1-\ln(\mathrm{e}+2)+\ln 3$ (i.e. dividing their correct final answer by 3) Otherwise, you can ignore incorrect working (isw) following on from a correct exact value.
	Note	A decimal answer of 1.641502724... (without a correct **exact** answer) is final A0
	Note	$\left[-3\ln(u+2)+3\ln u\right]_1^{\mathrm{e}}$ followed by awrt 1.64 (without a correct **exact** answer) is final M1A0

		Question 3 Notes Continued
3. (d)	**Note**	**BE CAREFUL! Candidates will assign their own "*A*" and "*B*" for this question.**
	Note	***Writing down*** $\dfrac{6}{(u+2)u}$ in the form $\dfrac{A}{(u+2)}+\dfrac{B}{u}$ with at least one of A or B correct is 1st M1
	Note	***Writing down*** $\dfrac{6}{(u+2)u}$ as $\dfrac{-3}{(u+2)}+\dfrac{3}{u}$ is 1st M1 1st A1.
	Note	**Condone** $\displaystyle\int\left(\frac{3}{u}-\frac{3}{(u+2)}\right)\mathrm{d}u$ to give $3\ln u-3\ln u+2$ (poor bracketing) for 2nd A1.
	Note	**Award M0A0M1A1ft** for a candidate who writes down e.g. $\displaystyle\int\frac{6}{u(u+2)}\,\mathrm{d}u=\int\left(\frac{6}{u}+\frac{6}{(u+2)}\right)\mathrm{d}u=6\ln u+6\ln(u+2)$ **AS EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ **AS PARTIAL FRACTIONS.**
	Note	**Award M0A0M0A0** for a candidate who writes down $\displaystyle\int\frac{6}{u(u+2)}\,\mathrm{d}u=6\ln u+6\ln(u+2)$ **or** $\displaystyle\int\frac{6}{u(u+2)}\,\mathrm{d}u=\ln u+6\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	**Award M1A1M1A1** for a candidate who writes down $\displaystyle\int\frac{6}{u(u+2)}\,\mathrm{d}u=3\ln u-3\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	If they lose the "6" and find $\displaystyle\int_1^{e}\frac{1}{u(u+2)}\mathrm{d}u$ we can allow a maximum of M1A0M1A1ftM1A0

	Question 3 Notes Continued		
3. (d) **Way 2**	$\left\{\displaystyle\int\frac{6}{u^2+2u}\mathrm{d}u=\int\frac{3(2u+2)}{u^2+2u}\mathrm{d}u-\int\frac{6u}{u^2+2u}\mathrm{d}u\right\}$		
	$=\displaystyle\int\frac{3(2u+2)}{u^2+2u}\mathrm{d}u-\int\frac{6}{u+2}\mathrm{d}u$	$\displaystyle\int\frac{\pm\alpha(2u+2)}{u^2+2u}\{\mathrm{d}u\}\pm\int\frac{\delta}{u+2}\{\mathrm{d}u\},\ \alpha,\beta,\delta\neq0$	M1
		Correct expression	A1
	$=3\ln(u^2+2u)-6\ln(u+2)$	Integrates $\dfrac{\pm M(2u+2)}{u^2+2u}\pm\dfrac{N}{u\pm k}$, $M,N,k\neq0$, to obtain any one of $\pm\lambda\ln(u^2+2u)$ **or** $\pm\mu\ln(\beta(u\pm k))$; $\lambda,\mu,\beta\neq0$	M1
		Integration of both terms is **correctly followed through** from **their** M and from **their** N	A1 ft
	$\left\{\text{So, }\left[3\ln(u^2+2u)-6\ln(u+2)\right]_1^{e}\right\}$ $=\left(3\ln(e^2+2e)-6\ln(e+2)\right)-\left(3\ln3-6\ln3\right)$	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$=3\ln(e^2+2e)-6\ln(e+2)+3\ln3$	$3\ln(e^2+2e)-6\ln(e+2)+3\ln3$	A1 o.e.
			[6]

3. (d) **Way 3**	Applying $u = \theta - 1$		
	$\left\{\int_1^e \frac{6}{u(u+2)}\,du = \right\} \int_2^{1+e} \frac{6}{(\theta-1)(\theta+1)}\,d\theta = \int_2^{1+e} \frac{6}{\theta^2-1}\,du = \left[3\ln\left(\frac{\theta-1}{\theta+1}\right)\right]_2^{1+e}$		M1A1M1A1
	$= 3\ln\left(\frac{1+e-1}{e+1+1}\right) - 3\ln\left(\frac{2-1}{2+1}\right) = 3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$	3rd M mark is dependent on 2nd M mark	dM1A1
			[6]

17.

Question Number	Scheme	Notes	Marks
5.	$y = e^x + 2e^{-x},\ x ^3 0$		
Way 1	$\{V =\}\ \pi \grave{\text{O}}_0^{\ln 4} \left(e^x + 2e^{-x}\right)^2 dx$	For $\pi \int \left(e^x + 2e^{-x}\right)^2$ Ignore limits and dx. Can be implied.	B1
	$= \{\pi\} \int_0^{\ln 4} \left(e^{2x} + 4e^{-2x} + 4\right) dx$	Expands $\left(e^x + 2e^{-x}\right)^2 \to \pm\alpha e^{2x} \pm \beta e^{-2x} \pm \delta$ where $\alpha, \beta, \delta \neq 0$. Ignore π, integral sign, limits and dx. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$	Integrates at least one of either $\pm\alpha e^{2x}$ to give $\pm\frac{\alpha}{2}e^{2x}$ or $\pm\beta e^{-2x}$ to give $\pm\frac{\beta}{2}e^{-2x}$ $\alpha, \beta ^1 0$	M1
		dependent on the 2nd M mark $e^{2x} + 4e^{-2x} \to \frac{1}{2}e^{2x} - 2e^{-2x}$, which can be simplified or un-simplified	A1
		$4 \to 4x$ or $4e^0x$	B1 cao
	$= \{\pi\}\left\{\left(\frac{1}{2}e^{2(\ln 4)} - 2e^{-2(\ln 4)} + 4(\ln 4)\right) - \left(\frac{1}{2}e^0 - 2e^0 + 4(0)\right)\right\}$	**dependent on the previous method mark.** Some evidence of applying limits of $\ln 4$ o.e. and 0 to a changed function in x and subtracts the correct way round. **Note:** A proper consideration of the limit of 0 is required.	dM1
	$= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$		

	$= \frac{75}{8}\pi + 4\pi\ln 4$ **or** $\frac{75}{8}\pi + 8\pi\ln 2$ **or** $\pi\left(\frac{75}{8} + 4\ln 4\right)$ **or** $\pi\left(\frac{75}{8} + 8\ln 2\right)$ **or** $\frac{75}{8}\pi + \ln 2^{8\pi}$ **or** $\frac{75}{8}\pi + \pi\ln 256$ **or** $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ **or** $\frac{1}{8}\pi(75 + 32\ln 4)$, etc		A1 isw
			[7]
			7

	Question 5 Notes	
5.	**Note**	π is only required for the 1st B1 mark and the final A1 mark.
	Note	Give 1st B0 for writing $\pi\int y^2\,dx$ followed by $2\pi\int\left(e^x + 2e^{-x}\right)^2 dx$
	Note	Give 1st M1 for $\left(e^x + 2e^{-x}\right)^2 \rightarrow e^{2x} + 4e^{-2x} + 2e^0 + 2e^0$ because $\delta = 2e^0 + 2e^0$
	Note	A decimal answer of 46.8731... or $\pi(14.9201...)$ (without a correct **exact** answer) is A0
	Note	$\pi\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$ followed by awrt 46.9 (without a correct **exact** answer) is final dM1A0
	Note	Allow exact equivalents which should be in the form $a\pi + b\pi\ln c$ or $\pi(a + b\ln c)$, where $a = \frac{75}{8}$ or $9\frac{3}{8}$ or 9.375. Do not allow $a = \frac{150}{16}$ or $9\frac{6}{16}$
	Note	Give B1M0M1A1B0M1A0 for the common response $\pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx \rightarrow \pi\int_0^{\ln 4}\left(e^{2x} + 4e^{-2x}\right)dx = \pi\left[\frac{1}{2}e^{2x} - 2e^{-2x}\right]_0^{\ln 4} = \frac{75}{8}\pi$

Question Number	Scheme	Notes	Marks
5. **Way 2**	$y = e^x + 2e^{-x},\ x \geq 0$		
	$\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$	For $\pi\int\left(e^x + 2e^{-x}\right)^2$ Ignore limits and dx. Can be implied.	B1
	$u = e^x \Rightarrow \frac{du}{dx} = e^x = u$ and $x = \ln 4 \Rightarrow u = 4, x = 0 \Rightarrow u = e^0 = 1$		
	$V = \{\pi\}\int_1^4\left(u + \frac{2}{u}\right)^2 \frac{1}{u}du = \{\pi\}\int_1^4\left(u^2 + \frac{4}{u^2} + 4\right)\frac{1}{u}du$		
	$= \{\pi\}\int_1^4\left(u + \frac{4}{u^3} + \frac{4}{u}\right)du$	$\left(e^x + 2e^{-x}\right)^2 \rightarrow \pm\alpha u \pm \beta u^{-3} \pm \delta u^{-1}$ where $u = e^x, \alpha, \beta, \delta \neq 0$. Ignore π, integral sign, limits and du. This can be implied by later work.	M1
	$= \{\pi\}\left[\frac{1}{2}u^2 - \frac{2}{u^2} + 4\ln u\right]_1^4$	Integrates at least one of either $\pm\alpha u$ to give $\pm\frac{\alpha}{2}u^2$ or $\pm\beta u^{-3}$ to give $\pm\frac{\beta}{2}u^{-2}$ $\alpha, \beta \neq 0$, where $u = e^x$	M1
		dependent on the 2nd M mark $u + 4u^{-3} \rightarrow \frac{1}{2}u^2 - 2u^{-2}$, simplified or un-simplified, where $u = e^x$	A1
		$4u^{-1} \rightarrow 4\ln u$, where $u = e^x$	B1 cao

	$=\{\pi\}\left\{\left(\frac{1}{2}(4)^2-\frac{2}{(4)^2}+4\ln4\right)-\left(\frac{1}{2}(1)^2-\frac{2}{(1)^2}+4\ln1\right)\right\}$	**dependent on the previous method mark.** Some evidence of applying limits of 4 and 1 to a changed function in u [or $\ln 4$ o.e. and 0 to an integrated function in x] and subtracts the correct way round.	dM1
	$=\{\pi\}\left(\left(8-\frac{1}{8}+4\ln4\right)-\left(\frac{1}{2}-2\right)\right)$		
	$=\frac{75}{8}\pi+4\pi\ln4$ **or** $\frac{75}{8}\pi+8\pi\ln2$ **or** $\pi\left(\frac{75}{8}+4\ln4\right)$ **or** $\pi\left(\frac{75}{8}+8\ln2\right)$ **or** $\frac{75}{8}\pi+\ln2^{8\pi}$ **or** $\frac{75}{8}\pi+\pi\ln256$ **or** $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ **or** $\frac{1}{8}\pi(75+32\ln4)$, etc		A1 isw
			[7]

18.

Question Number	Scheme		Notes	Marks
7.	$\frac{dh}{dt}=k\sqrt{(h-9)},\ 9<h\le200;\quad h=130,\ \frac{dh}{dt}=-1.1$			
(a)	$-1.1=k\sqrt{(130-9)}\ \Rightarrow\ k=...$		Substitutes $h=130$ and either $\frac{dh}{dt}=-1.1$ or $\frac{dh}{dt}=1.1$ into the printed equation and rearranges to give $k=...$	M1
	so, $k=-\frac{1}{10}$ or -0.1		$k=-\frac{1}{10}$ or -0.1	A1
				[2]
(b) **Way 1**	$\int\frac{dh}{\sqrt{(h-9)}}=\int k\,dt$		Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs.	B1
	$\int(h-9)^{-\frac{1}{2}}\,dh=\int k\,dt$			
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}=kt\ (+c)$		Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda,\mu\ne0$	M1
			$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}=kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}=$ (their k)t, with/without $+c$, or equivalent, which can be un-simplified or simplified.	A1
	$\{t=0,h=200\Rightarrow\}\ 2\sqrt{(200-9)}=k(0)+c$		Some evidence of applying both $t=0$ and $h=200$ to changed equation containing a constant of integration, e.g. c or A	M1
	$\Rightarrow c=2\sqrt{191}\ \Rightarrow 2(h-9)^{\frac{1}{2}}=-0.1t+2\sqrt{191}$ $\{h=50\Rightarrow\}\ 2\sqrt{(50-9)}=-0.1t+2\sqrt{191}$ $t=...$		**dependent on the previous M mark** Applies $h=50$ and their value of c to their changed equation and rearranges to find the value of $t=...$	dM1
	$t=20\sqrt{191}-20\sqrt{41}$ or $t=148.3430145...=148$ (minutes) (nearest minute)		$t=20\sqrt{191}-20\sqrt{41}$ isw or awrt 148	A1 cso
				[6]

(b) **Way 2**	$\displaystyle\int_{200}^{50} \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int_{0}^{T} k \, \mathrm{d}t$	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary.	B1
	$\displaystyle\int_{200}^{50} (h-9)^{-\frac{1}{2}} \mathrm{d}h = \int_{0}^{T} k \, \mathrm{d}t$		
	$\displaystyle\left[\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{200}^{50} = [kt]_0^T$	Integrates $\dfrac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\dfrac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\dfrac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} =$ (their k)t, with/without limits, or equivalent, which can be un-simplified or simplified.	A1
	$2\sqrt{41} - 2\sqrt{191} = kt$ or kT	Attempts to apply limits of $h = 200$, $h = 50$ and (can be implied) $t = 0$ to their changed equation	M1
	$t = \dfrac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$	**dependent on the previous M mark** Then rearranges to find the value of $t = \ldots$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148 **or** 2 hours and awrt 28 minutes	A1 cso
			[6]
			8

		Question 7 Notes
7. (b)	**Note**	Allow first B1 for writing $\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{k\sqrt{(h-9)}}$ or $\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{(\text{their } k)\sqrt{(h-9)}}$ or equivalent
	Note	$\dfrac{\mathrm{d}t}{\mathrm{d}h} = \dfrac{1}{k\sqrt{(h-9)}}$ leading to $t = \dfrac{2}{k}\sqrt{(h-9)}\ (+\,c)$ with/without $+\,c$ is B1M1A1
	Note	After finding $k = 0.1$ in part (a), it is only possible to gain full marks in part (b) by **initially writing** $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{(h-9)}$ **or** $\displaystyle\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int -k\,\mathrm{d}t$ **or** $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -0.1\sqrt{(h-9)}$ **or** $\displaystyle\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int -0.1\,\mathrm{d}t$ Otherwise, those candidates who find $k = 0.1$ in part (a), should lose at least the final A1 mark in part (b).

Question Number	Scheme	Notes	Marks
8.	$x = 3\theta\sin\theta,\ \ y = \sec^3\theta,\ \ 0 \le \theta < \frac{\pi}{2}$		
(a)	$\{\text{When } y=8,\}\ 8 = \sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	Sets $y=8$ to find θ **and** attempts to substitute their θ into $x = 3\theta\sin\theta$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		**[2]**
(b)	$\frac{dx}{d\theta} = 3\sin\theta + 3\theta\cos\theta$	$3\theta\sin\theta \to 3\sin\theta + 3\theta\cos\theta$ Can be implied by later working	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\} = \int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{d\theta\}$	Applies $(\pm K\sec^3\theta)\left(\text{their } \frac{dx}{d\theta}\right)$ Ignore integral sign and $d\theta$; $K \neq 0$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\,d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. **Must have** integral sign and $d\theta$ in their final answer.	A1 *
	$x=0$ and $x=k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	$\alpha = 0$ and $\beta = \frac{\pi}{3}$ or evidence of $0 \to 0$ and $k \to \frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		**[4]**
(c) **Way 1**	$\left\{\int \theta\sec^2\theta\,d\theta\right\} = \theta\tan\theta - \int\tan\theta\{d\theta\}$	$\theta\sec^2\theta \to A\theta g(\theta) - B\int g(\theta),\ A>0,\ B>0,$ where $g(\theta)$ is a trigonometric function in θ **and** $g(\theta) = $ their $\int\sec^2\theta\,d\theta$. [Note: $g(\theta) \neq \sec^2\theta$]	M1
		dependent on the previous M mark **Either** $\lambda\theta\sec^2\theta \to A\theta\tan\theta - B\int\tan\theta,\ A>0,\ B>0$ **or** $\theta\sec^2\theta \to \theta\tan\theta - \int\tan\theta$	dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec\theta)$ or $\theta\tan\theta + \ln(\cos\theta)$ **or** $\lambda\theta\sec^2\theta \to \lambda\theta\tan\theta - \lambda\ln(\sec\theta)$ or $\lambda\theta\tan\theta + \lambda\ln(\cos\theta)$	A1
	Note: Condone $\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec x)$ or $\theta\tan\theta + \ln(\cos x)$ for A1		
	$\left\{\int\tan\theta\sec^2\theta\,d\theta\right\}$	$\tan\theta\sec^2\theta$ or $\lambda\tan\theta\sec^2\theta \to \pm C\tan^2\theta$ or $\pm C\sec^2\theta$ or $\pm Cu^{-2}$, where $u = \cos\theta$	M1
	$= \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u = \cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	$\tan\theta\sec^2\theta \to \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2\cos^2\theta}$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ or $0.5u^{-2}$, where $u = \cos\theta$ or $0.5u^2$, where $u = \tan\theta$ or $\lambda\tan\theta\sec^2\theta \to \frac{\lambda}{2}\tan^2\theta$ or $\frac{\lambda}{2}\sec^2\theta$ or $\frac{\lambda}{2\cos^2\theta}$ or $0.5\lambda u^{-2}$, where $u = \cos\theta$ or $0.5\lambda u^2$, where $u = \tan\theta$	A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$		
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$		
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$		A1 o.e.
			[6]
			12

Question Number	Scheme	Notes	Marks
8. (c) **Way 2**	**Way 2 for the first 5 marks:** Applying integration by parts on $\int(\theta+\tan\theta)\sec^2\theta\,\mathrm{d}\theta$		
	$\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\,\mathrm{d}\theta = \int(\theta+\tan\theta)\sec^2\theta\,\mathrm{d}\theta,$	$\left\{\begin{array}{l} u=\theta+\tan\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta}=1+\sec^2\theta \\ \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec^2\theta \quad \Rightarrow v=\tan\theta=\mathrm{g}(\theta)\end{array}\right\}$	
	$\mathrm{h}(\theta)$ and $\mathrm{g}(\theta)$ are trigonometric functions in θ **and** $\mathrm{g}(\theta)=$ their $\int\sec^2\theta\,\mathrm{d}\theta$. [**Note:** $\mathrm{g}(\theta)\neq\sec^2\theta$]		
	$=(\theta+\tan\theta)\tan\theta-\int(1+\sec^2\theta)\tan\theta\{\mathrm{d}\theta\}$	$A(\theta+\tan\theta)\mathrm{g}(\theta)-B\int(1+\mathrm{h}(\theta))\mathrm{g}(\theta),\ A>0,\ B>0$	M1
		dependent on the previous M mark **Either** $\lambda\left[(\theta+\tan\theta)\sec^2\theta\right]\to$ $A(\theta+\tan\theta)\tan\theta-B\int(1+\mathrm{h}(\theta))\tan\theta,\ A\neq 0,\ B>0$ **or** $(\theta+\tan\theta)\tan\theta-\int(1+\mathrm{h}(\theta))\tan\theta$	dM1
	$=(\theta+\tan\theta)\tan\theta-\int(\tan\theta+\tan\theta\sec^2\theta)\{\mathrm{d}\theta\}$		
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\int\tan\theta\sec^2\theta\{\mathrm{d}\theta\}$	$(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)$ o.e. or $\lambda\left[(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)\right]$ o.e.	A1
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\tan^2\theta$	$\tan\theta\sec^2\theta\to\pm C\tan^2\theta$ or $\pm C\sec^2\theta$	M1
	or $=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\sec^2\theta$ etc.	$(\theta+\tan\theta)\tan\theta-\frac{1}{2}\tan^2\theta$ or $(\theta+\tan\theta)\tan\theta-\frac{1}{2}\sec^2\theta$	A1
	Note	Allow the first two marks in part (c) for $\theta\tan\theta-\int\tan\theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta\tan\theta-\ln(\sec\theta)$ embedded in their working	
	Note	Allow 3rd M1 2nd A1 marks for either $\tan^2\theta-\frac{1}{2}\tan^2\theta$ or $\tan^2\theta-\frac{1}{2}\sec^2\theta$ embedded in their working	
	Question 8 Notes		
8. (a)	**Note**	Allow M1 for an answer of $k=$ awrt 2.72 without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	Allow M1 for an answer of $k=3\left(\arccos(\tfrac{1}{2})\right)\sin\left(\arccos(\tfrac{1}{2})\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	E.g. allow M1 for $\theta=60^\circ$, leading to $k=3(60)\sin(60)$ or $k=90\sqrt{3}$	

	Question 8 Notes Continued	
8. (b)	**Note**	To gain A1, $\mathrm{d}\theta$ does not need to appear until they obtain $3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$
	Note	For M1, their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$, where their $\frac{\mathrm{d}x}{\mathrm{d}\theta}\neq 3\theta\sin\theta$, needs to be a trigonometric function in θ
	Note	Writing $\int(\sec^3\theta)(3\sin\theta+3\theta\cos\theta)$ = $3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	Writing $\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\sin\theta+3\theta\cos\theta$ followed by writing $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta=3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	The final A mark would be lost for $\int\frac{1}{\cos^3\theta}3\sin\theta+3\theta\cos\theta$ = $3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ [lack of brackets in this particular case].
	Note	Give 2nd B0 for $\alpha=0$ and $\beta=60°$, without reference to $\beta=\frac{\pi}{3}$
(c)	**Note**	A decimal answer of 7.861956551... (without a correct **exact** answer) is A0.
	Note	First three marks are for integrating $\theta\sec^2\theta$ with respect to θ
	Note	Fourth and fifth marks are for integrating $\tan\theta\sec^2\theta$ with respect to θ
	Note	Candidates are not penalised for writing $\ln\lvert\sec\theta\rvert$ as either $\ln(\sec\theta)$ or $\ln\sec\theta$
	Note	$\theta\sec^2\theta\to\theta\tan\theta+\ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta\to\theta\tan\theta-\ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta\to\theta\tan\theta-\ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	$\theta\sec^2\theta\to\theta\tan\theta+\ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	Writing a correct $uv-\int v\frac{\mathrm{d}u}{\mathrm{d}x}$ with $u=\theta$, $\frac{\mathrm{d}v}{\mathrm{d}\theta}=\tan\theta$, $\frac{\mathrm{d}u}{\mathrm{d}\theta}=1$ and $v=$ their $g(\theta)$ and making one error in the direct application of this formula is 1st M1 only.

8. (c)	Alternative method for finding $\int\tan\theta\sec^2\theta\,\mathrm{d}\theta$		
	$\left\{u=\tan\theta\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}\theta}=\sec^2\theta;\ \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec^2\theta\Rightarrow v=\tan\theta\right\}$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\tan^2\theta-\int\tan\theta\sec^2\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\tan^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\frac{1}{2}\tan^2\theta$	$\tan\theta\sec^2\theta$ or $\to\pm C\tan^2\theta$	M1
		$\tan\theta\sec^2\theta\to\frac{1}{2}\tan^2\theta$	A1
	or $\left\{u=\sec\theta\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}\theta}=\sec\theta\tan\theta;\ \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec\theta\tan\theta\Rightarrow v=\sec\theta\right\}$		
	$\Rightarrow\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\sec^2\theta-\int\sec^2\theta\tan\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\sec^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\frac{1}{2}\sec^2\theta$	$\tan\theta\sec^2\theta$ or $\to\pm C\sec^2\theta$	M1
		$\tan\theta\sec^2\theta\to\frac{1}{2}\sec^2\theta$	A1

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20.

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>2.</td><td>

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625	***0.659485...***	1.2032	1.9044	2.7726

</td><td>$y = x^2 \ln x$</td><td></td></tr>
<tr><td>(a)</td><td>$\{\text{At } x = 1.4,\}\ y = 0.6595 \text{ (4 dp)}$</td><td>0.6595</td><td>B1 cao</td></tr>
<tr><td></td><td colspan="2"></td><td>[1]</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2} \times (0.2) \times \left[0 + 2.7726 + 2\left(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044\right)\right]$</td><td>Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$</td><td>B1 o.e.</td></tr>
<tr><td></td><td>{Note: The "0" does not have to be included in [......]}</td><td>For structure of [..............]</td><td>M1</td></tr>
<tr><td></td><td>$\left\{= \frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083 \text{ (3 dp)}$</td><td>anything that rounds to 1.083</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>[3]</td></tr>
<tr><td>(c)
Way 1</td><td>$\left\{\mathrm{I} = \int x^2 \ln x \, \mathrm{d}x\right\}$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{array}\right\}$</td><td></td><td></td></tr>
<tr><td></td><td rowspan="2">$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$</td><td>Either $x^2 \ln x \to \pm \lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{\mathrm{d}x\}$
or $\pm \lambda x^3 \ln x - \int \mu x^2 \{\mathrm{d}x\}$, where $\lambda, \mu > 0$</td><td>M1</td></tr>
<tr><td></td><td>$x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$,
simplified or un-simplified</td><td>A1</td></tr>
<tr><td></td><td>$= \frac{x^3}{3}\ln x - \frac{x^3}{9}$</td><td>$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified</td><td>A1</td></tr>
<tr><td></td><td>$\text{Area}(R) = \left\{\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2\right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$</td><td>dependent on the previous M mark. Applies limits of 2 and 1 and subtracts the correct way round</td><td>dM1</td></tr>
<tr><td></td><td>$= \frac{8}{3}\ln 2 - \frac{7}{9}$</td><td>$\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$</td><td>A1 oe cso</td></tr>
<tr><td></td><td colspan="2"></td><td>[5]</td></tr>
</table>

(c) **Way 2**	$\text{I} = x^2(x\ln x - x) - \int 2x(x\ln x - x)\,dx$	$\left\{\begin{array}{ll} u = x^2 & \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = \ln x & \Rightarrow v = x\ln x - x \end{array}\right\}$	
	So, $3\text{I} = x^2(x\ln x - x) + \int 2x^2\{dx\}$		
	and $\text{I} = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$	A full method of applying $u = x^2,\ v' = \ln x$ to give $\pm\lambda x^2(x\ln x - x) \pm \mu\int x^2\{dx\}$	M1
		$\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{dx\}$ simplified or un-simplified	A1
	$= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]
			9

		Question 2 Notes
2. (a)	**B1**	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	M1	<u>For structure of trapezium rule</u>[<u>............</u>]
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 1.083
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…)
	Note	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	Note	Award B1M1A1 for $\frac{1}{10}(2.7726) + \frac{1}{5}(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) = \text{awrt } 1.083$
		<u>Bracketing mistake:</u> Unless the final answer implies that the calculation has been done correctly
		Award B1M0A0 for $\frac{1}{2}(0.2) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044) + 2.7726$ (answer of 10.9318)
		Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726) + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)$ (answer of 8.33646)
		<u>Alternative method: Adding individual trapezia</u>
		$\text{Area} \approx 0.2\times\left[\frac{0+0.2625}{2} + \frac{0.2625+"0.6595"}{2} + \frac{"0.6595"+1.2032}{2} + \frac{1.2032+1.9044}{2} + \frac{1.9044+2.7726}{2}\right] = 1.08318...$
	B1	0.2 and a divisor of 2 on all terms inside brackets
	M1	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2
	A1	anything that rounds to 1.083

(c)	A1	Exact answer needs to be a two term expression in the form $a\ln b + c$
	Note	Give A1 e.g. $\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$ or $\frac{4}{3}\ln 4 - \frac{7}{9}$ or $\frac{1}{3}\ln 256 - \frac{7}{9}$ or $-\frac{7}{9} + \frac{8}{3}\ln 2$ or $\ln 2^{\frac{8}{3}} - \frac{7}{9}$ or equivalent.
	Note	Give final A0 for a final answer of $\frac{8\ln 2 - \ln 1}{3} - \frac{7}{9}$ or $\frac{8\ln 2}{3} - \frac{1}{3}\ln 1 - \frac{7}{9}$ or $\frac{8\ln 2}{3} - \frac{8}{9} + \frac{1}{9}$ or $\frac{8}{3}\ln 2 - \frac{7}{9} + c$
	Note	$\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	Note	Give dM0A0 for $\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2 \to \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \frac{1}{9}$ (adding rather than subtracting)
	Note	Allow dM1A0 for $\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2 \to \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 + \frac{1}{9}\right)$
	SC	A candidate who uses $u = \ln x$ and $\frac{dv}{dx} = x^2$, $\frac{du}{dx} = \frac{\alpha}{x}$, $v = \beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1.

21.

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,dx = \int -\frac{5}{2}\,dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $\ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow$ $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 cso
			[4]

(a) **Way 2**	$\dfrac{dt}{dx} = -\dfrac{2}{5x}$ or $t = \displaystyle\int -\dfrac{2}{5x}\,dx$	**Either** $\dfrac{dt}{dx} = -\dfrac{2}{5x}$ **or** $t = \displaystyle\int -\dfrac{2}{5x}\,dx$	B1
	$t = -\dfrac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = ...$ **or** $\pm\alpha\ln px\,;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\dfrac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ c = \dfrac{2}{5}\ln 60 \Rightarrow t = -\dfrac{2}{5}\ln x + \dfrac{2}{5}\ln 60$ $\Rightarrow -\dfrac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \dfrac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 3**	$\displaystyle\int_{60}^{x}\dfrac{1}{x}\,dx = \int_{0}^{t} -\dfrac{5}{2}\,dt$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\dfrac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\dfrac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\dfrac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \dfrac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]
(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\dfrac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda e^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\dfrac{2}{5}\ln\left(\dfrac{20}{60}\right)$ $\{= 0.4394449...\ \text{(days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$ □ $, t > 0$)	dM1
	$\Rightarrow t = 632.8006... = 633$ (to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 **cso**
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $\frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with **no incorrect working seen**	A1 **cso**
			[4]
(a) **Way 5**	$\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}$ $t = \int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 **cso**
			[4]

		Question 4 Notes
4. (a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \rightarrow x = 60e^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.

(b)	A1	You can apply **cso** for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

22.

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)}\,dy,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx,\ x = 4\sin^2\theta$		
(i) **Way 1**	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 \equiv A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow -4 = 2A \Rightarrow A = -2$	At least one of their $A = -2$ **or** their $B = 9$	A1
	$y = -\frac{2}{3} \Rightarrow -6 = -\frac{2}{3}B \Rightarrow B = 9$	Both their $A = -2$ **and** their $B = 9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{-2}{y} + \frac{9}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{A}{y} \to \pm\lambda \ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $A \neq 0,\ B \neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y + \frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 **cao**
			[6]
(ii) (a) **Way 1**	$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{dx}{d\theta} = 8\sin\theta\cos\theta$ or $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 8\sin\theta\cos\theta\,d\theta$		B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{d\theta\}$ or $\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,4\sin 2\theta\ \{d\theta\}$		M1
	$= \int \underline{\underline{\tan\theta}}\,.\,8\sin\theta\cos\theta\ \{d\theta\}$ or $\int \underline{\underline{\tan\theta}}\,.\,4\sin 2\theta\ \{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$= \int 8\sin^2\theta\,d\theta$	$\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1
	$3 = 4\sin^2\theta$ **or** $\frac{3}{4} = \sin^2\theta$ **or** $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x = 0 \to \theta = 0\}$	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]

(ii) (b)	$= \{8\}\int\left(\frac{1-\cos 2\theta}{2}\right)\mathrm{d}\theta \quad \left\{= \int(4-4\cos 2\theta)\mathrm{d}\theta\right\}$	Applies $\cos 2\theta = 1 - 2\sin^2\theta$ to their integral. (See notes)	M1
	$= \{8\}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right) \quad \{= 4\theta - 2\sin 2\theta\}$	For $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha,\beta \neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0+0)\right)$		
	$= \frac{4}{3}\pi - \sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}\left(4\pi - 3\sqrt{3}\right)$	A1 o.e.
			[4]
			15

	Question 6 Notes	
6. (i)	**1st M1**	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied ***for writing down*** either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
6. (ii)(a)	**1st M1**	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{\left(\frac{x}{4-x}\right)}\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3, \theta = \frac{\pi}{3}; x = 0, \theta = 0$

(ii)(b)	M1	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\frac{1-\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A ***correct solution in part (ii)*** leading to a "two term" exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397... (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,d\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
6. (i) Way 2	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$		
	$\frac{3y+6}{y(3y+2)} = \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 **cao**
			[6]

6. (i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Scheme	Notes	
6. (i) **Way 4**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y}{y(3y+2)}\,dy - \int \frac{4}{y(3y+2)}\,dy$		
	$= \int \frac{3}{(3y+2)}\,dy - \int \frac{4}{y(3y+2)}\,dy$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3}{3y+2}\,dy - \int \frac{2}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0, B \neq 0, C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) **Way 2**	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \dfrac{dx}{d\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\int\sqrt{\dfrac{4\sin^2\theta}{4-4\sin^2\theta}}.\,8\sin\theta\cos\theta\,\{d\theta\}$	As before	M1
	$= \int\sqrt{\dfrac{\sin^2\theta}{(1-\sin^2\theta)}}.\,8\cos\theta\sin\theta\,\{d\theta\}$		
	$= \int\dfrac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\,\{d\theta\}$		
	$= \int\sin\theta.\,8\sin\theta\,\{d\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$= \int 8\sin^2\theta\,d\theta$	$\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1 **cso**
(ii)(a) **Way 3**	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \dfrac{dx}{d\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta\,,\;\; 4 - x = 2 + 2\cos 2\theta$		
	$\int\sqrt{\dfrac{2-2\cos 2\theta}{2+2\cos 2\theta}}.\,4\sin 2\theta\,\{d\theta\}$		M1
	$= \int\dfrac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}.\dfrac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}4\sin 2\theta\,\{d\theta\} = \int\dfrac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}.\,4\sin 2\theta\,\{d\theta\}$		
	$= \int\dfrac{2-2\cos 2\theta}{2\sin 2\theta}.\,4\sin 2\theta\,\{d\theta\} = \int 2(2-2\cos 2\theta).\,\{d\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$= \int 8\sin^2\theta\,d\theta$	$\int 8\sin^2\theta\,d\theta$ including $d\theta$	A1 **cso**

23.

Question Number	Scheme	Notes	Marks
7.	$y = (2x-1)^{\frac{3}{4}}\,,\quad x \geqslant \dfrac{1}{2}$ passes though $P(k, 8)$		
(a)	$\left\{\int (2x-1)^{\frac{3}{2}}\,dx\right\} = \dfrac{1}{5}(2x-1)^{\frac{5}{2}}\,\{+c\}$	$(2x \pm 1)^{\frac{3}{2}} \to \pm\lambda(2x \pm 1)^{\frac{5}{2}}$ **or** $\pm\lambda u^{\frac{5}{2}}$ where $u = 2x \pm 1$; $\lambda \neq 0$	M1
		$\dfrac{1}{5}(2x-1)^{\frac{5}{2}}$ with or without $+c$. Must be simplified.	A1
			[2]
(b)	$\{P(k, 8) \Rightarrow\}\; 8 = (2k-1)^{\frac{3}{4}} \Rightarrow k = \dfrac{8^{\frac{4}{3}}+1}{2}$	Sets $8 = (2k-1)^{\frac{3}{4}}$ or $8 = (2x-1)^{\frac{3}{4}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k = \dfrac{17}{2}$	k (or x) $= \dfrac{17}{2}$ or 8.5	A1
			[2]

(c)	$\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2 \,dx$	For $\pi\int\left((2x-1)^{\frac{3}{4}}\right)^2$ or $\pi\int(2x-1)^{\frac{3}{2}}$ Ignore limits and dx. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}} y^2\,dx\right\} = \left[\frac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}} = \left(\left(\frac{16^{\frac{5}{2}}}{5}\right)-(0)\right) \left\{=\frac{1024}{5}\right\}$ **Note:** It is not necessary to write the "-0"	Applies x-limits of "8.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^{\frac{5}{2}}$; $\beta \neq 0$ and subtracts the correct way round.	M1
	$\{V_{\text{cylinder}}\} = \pi(8)^2\left(\frac{17}{2}\right) \{= 544\pi\}$	$\pi(8)^2$(their answer to part (b)) $V_{\text{cylinder}} = 544\pi$ implies this mark	B1 ft
	$\left\{\text{Vol}(S) = 544\pi - \frac{1024\pi}{5}\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$	An exact correct answer in the form $k\pi$ E.g. $\frac{1696}{5}\pi$, $\frac{3392}{10}\pi$ or 339.2π	A1
			[4]
Alt. (c)	$\text{Vol}(S) = \pi(8)^2\left(\frac{1}{2}\right) + \pi\int_{0.5}^{8.5}\left(8^2 - (2x-1)^{\frac{3}{2}}\right)dx$	For $\pi\int .....(2x-1)^{\frac{3}{2}}$ Ignore limits and dx.	B1
	$= \pi(8)^2\left(\frac{1}{2}\right) + \pi\left[64x - \frac{1}{5}(2x-1)^{\frac{5}{2}}\right]_{0.5}^{8.5}$		
	$= \pi(8)^2\left(\frac{1}{2}\right) + \pi\left(\left(64(\text{"}8.5\text{"}) - \frac{1}{5}(2(8.5)-1)^{\frac{5}{2}}\right) - \left(64(0.5) - \frac{1}{5}(2(0.5)-1)^{\frac{5}{2}}\right)\right)$	as above	M1 B1
	$\left\{= 32\pi + \pi\left(\left(544 - \frac{1024}{5}\right) - (32-0)\right)\right\} \Rightarrow \text{Vol}(S) = \frac{1696}{5}\pi$		A1
			[4]
			8

		Question 7 Notes
7. (b)	SC	Allow Special Case SC M1 for a candidate who sets $8 = (2k-1)^{\frac{3}{2}}$ or $8 = (2x-1)^{\frac{3}{2}}$ and rearranges to give $k =$ (or $x =$) a numerical value.
7. (c)	M1	Can also be given for applying u-limits of "16" (2("part (b)") − 1) and 0 to an expression of the form $\pm\beta u^{\frac{5}{2}}$; $\beta \neq 0$ and subtracts the correct way round.
	Note	You can give M1 for $\left[\frac{(2x-1)^{\frac{5}{2}}}{5}\right]_{\frac{1}{2}}^{\frac{17}{2}} = \frac{1024}{5}$
	Note	Give M0 for $\left[\frac{(2x-1)^{\frac{5}{2}}}{5}\right]_{0}^{\frac{17}{2}} = \left(\left(\frac{16^{\frac{5}{2}}}{5}\right)-(0)\right)$
	B1ft	Correct expression for the volume of a cylinder with radius 8 and their (part (b)) height k.
	Note	If a candidate uses integration to find the volume of this cylinder they need to apply their limits to give a correct expression for its volume. So $\pi\int_0^{8.5} 8^2\,dx = \pi\left[64x\right]_0^{8.5}$ is **not sufficient** for B1 but $\pi(64(8.5)-0)$ **is sufficient** for B1.

7.	MISREADING IN BOTH PARTS (B) AND (C)		
	Apply the misread rule (MR) for candidates who apply $y=(2x-1)^{\frac{3}{2}}$ to **both** parts (b) and (c)		
(b)	$\{P(k,8)\Rightarrow\}\ 8=(2k-1)^{\frac{3}{2}}\Rightarrow k=\dfrac{8^{\frac{2}{3}}+1}{2}$	Sets $8=(2k-1)^{\frac{3}{2}}$ or $8=(2x-1)^{\frac{3}{2}}$ and rearranges to give $k=$ (or $x=$) a numerical value.	M1
	So, $k=\dfrac{5}{2}$	k (or x) $=\dfrac{5}{2}$ or 2.5	A1
			[2]
(c)	$\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2 \mathrm{d}x$	For $\pi\int\left((2x-1)^{\frac{3}{2}}\right)^2$ or $\pi\int(2x-1)^3$ Ignore limits and dx. Can be implied.	B1
	$\left\{\int_{\frac{1}{2}}^{\frac{17}{2}} y^2\,\mathrm{d}x\right\}=\left[\dfrac{(2x-1)^4}{8}\right]_{\frac{1}{2}}^{\frac{5}{2}}=\left(\left(\dfrac{4^4}{8}\right)-(0)\right)\ \{=32\}$	Applies x-limits of "2.5" (their answer to part (b)) and 0.5 to an expression of the form $\pm\beta(2x-1)^4;\ \beta\neq 0$ and subtracts the correct way round.	M1
	$V_{\text{cylinder}}=\pi(8)^2\left(\dfrac{5}{2}\right)\ \{=160\pi\}$	$\pi(8)^2$(their answer to part (b)) Sight of 160π implies this mark	B1 ft
	$\{\mathrm{Vol}(S)=160\pi-32\pi\}\Rightarrow \mathrm{Vol}(S)=128\pi$	An exact correct answer in the form $k\pi$ E.g. 128π	A1
			[4]
	Note	Mark parts (b) and (c) using the mark scheme above and then working forwards from part (b) deduct two from any A or B marks gained. E.g. (b) M1A1 (c) B1M1B1A1 would score (b) M1A0 (c) B0M1B1A1 E.g. (b) M1A1 (c) B1M1B0A0 would score (b) M1A0 (c) B0M1B0A0	
	Note	If a candidate uses $y=(2x-1)^{\frac{3}{4}}$ in part (b) and then uses $y=(2x-1)^{\frac{3}{2}}$ in part (c) do not apply a misread in part (c).	

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24.

Question Number	Scheme		Marks
3.	$y=4x-x\mathrm{e}^{\frac{1}{2}x},\ x\geqslant 0$		
(a)	$\left\{y=0\Rightarrow 4x-x\mathrm{e}^{\frac{1}{2}x}=0\Rightarrow x(4-\mathrm{e}^{\frac{1}{2}x})=0\Rightarrow\right\}$		
	$\mathrm{e}^{\frac{1}{2}x}=4\Rightarrow x_A=4\ln 2$	Attempts to solve $\mathrm{e}^{\frac{1}{2}x}=4$ giving $x=\ldots$ in terms of $\pm\lambda\ln\mu$ where $\mu>0$	M1
		$4\ln 2$ **cao** (Ignore $x=0$)	A1
			[2]
(b)	$\left\{\int x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x\right\}=2x\mathrm{e}^{\frac{1}{2}x}-\int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$	$\alpha x\mathrm{e}^{\frac{1}{2}x}-\beta\int\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\},\ \alpha>0,\beta>0$	M1
		$2x\mathrm{e}^{\frac{1}{2}x}-\int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, with or without d$x$	A1 (M1 on ePEN)
	$=2x\mathrm{e}^{\frac{1}{2}x}-4\mathrm{e}^{\frac{1}{2}x}\ \{+c\}$	$2x\mathrm{e}^{\frac{1}{2}x}-4\mathrm{e}^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]

(c)	$\left\{\int 4x\,\mathrm{d}x\right\} = 2x^2$ $4x \to 2x^2$ or $\frac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x\right\} = \left[2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$	
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 4\mathrm{e}^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)\mathrm{e}^{\frac{1}{2}(0)} + 4\mathrm{e}^{\frac{1}{2}(0)}\right)$ **See notes**	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$ $32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
		[3] **8**

		Question 3 Notes
3. (a)	**M1**	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	**NOTE**	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, where $\alpha > 0, \beta > 0$. (must be in this form) with or without $\mathrm{d}x$
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$ or equivalent, with or without $\mathrm{d}x$. **Can be un-simplified.**
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ or equivalent with or without $+ c$. **Can be un-simplified.**
	Note	You can also allow $2\mathrm{e}^{\frac{1}{2}x}(x-2)$ or $\mathrm{e}^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	<u>**SPECIAL CASE:**</u> A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

3. (c)	**B1**	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \dfrac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	$5.19378\ldots$ without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	$5.19378\ldots$ following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.
	Note	$5.19378\ldots$ from no working is M0A0.

25.

Question Number	Scheme		Marks
6. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\,dx$, $x = 1 + 2\sin\theta$		
	$\dfrac{dx}{d\theta} = 2\cos\theta$	$\dfrac{dx}{d\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\,dx \text{ or } \int \sqrt{(3+2x-x^2)}\,dx\right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\;2\cos\theta\,\{d\theta\}$	Substitutes for both x and dx, where $dx \neq \lambda d\theta$. Ignore $d\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\;2\cos\theta\,\{d\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\;2\cos\theta\,\{d\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\;2\cos\theta\,\{d\theta\}$ or $\int \sqrt{4\cos^2\theta}\;2\cos\theta\,\{d\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\,d\theta$, $\{k = 4\}$	$4\int \cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$ **Note:** $d\theta$ is required here.	A1

	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta \{d\theta\}\right\} = \{k\}\int\left(\frac{1+\cos 2\theta}{2}\right)\{d\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 **(A1 on ePEN)**
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 **cao cso**
			[3] **8**

		Question 6 Notes
6. (a)	**B1**	$\frac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta\, d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged } \frac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\, dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int\sqrt{(3-x)(x+1)}\, dx$ is equal to $4\int\cos^2\theta\, d\theta$ or $\int 4\cos^2\theta\, d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.

	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\frac{\pi}{6};\ x = 3,\ \theta = \frac{\pi}{2}$
(b)	**NOTE**	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\frac{1 + \cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta),\ \alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A ***correct solution in part (b)*** leading to a "two term" exact answer. Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

26.

Question Number	Scheme		Marks
7. (a)	$\frac{2}{P(P-2)} \equiv \frac{A}{P} + \frac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\frac{1}{(P-2)} - \frac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t$		
	$\int \frac{2}{P(P-2)}\, dP = \int \cos 2t\, dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0, \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$	A1
	$\{t = 0, P = 3 \Rightarrow\}$ $\ln 1 - \ln 3 = 0 + c$ $\{\Rightarrow c = -\ln 3$ or $\ln(\frac{1}{3})\}$	See notes	M1

	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t - \ln 3$ $\ln\left(\frac{3(P-2)}{P}\right) = \frac{1}{2}\sin 2t$		
	$\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\frac{1}{2}\sin 2t = \ln\left(\frac{3(4-2)}{4}\right)\ \left\{= \ln\left(\frac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] 13

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
7. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t\ \ (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\}\quad 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

		Question 7 Notes
7. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\dfrac{1}{(P-2)} - \dfrac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\dfrac{A}{P} + \dfrac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\dfrac{1}{(P-2)} - \dfrac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1,\ B = 1$

7. (b)	**B1**	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\displaystyle\int \frac{2}{P^2 - 2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\displaystyle\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ **o.e.** are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP$; M, N can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2 - 2P)$ or $\ln(P^2 - 2P)$
	1st A1	Correct result of $\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\dfrac{(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t + c$ followed by $\dfrac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\dfrac{(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t + c \rightarrow \dfrac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c} \rightarrow \dfrac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is final M1M0A0

		4th M1 for making *P* the subject **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** **(1)** M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t} \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ **(2)** M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - e^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ **(3)** M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3e^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3e^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2}\sin 2t}$ leading to $P = ..$
(c)	**M1**	States $P = 4$ or applies $P = 4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. ***(Do not apply isw here)***
	Note	***Do not apply ignore subsequent working for A1.*** (Eg: 0.473 followed by 473 years is A0.)
	Note	Use of $P = 4000$: Without the mention of $P = 4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	Use of Degrees: $t =$ awrt 27.1 will usually imply M1M1A0

27.

Question Number	Scheme		Marks
8. (a)	$\left\{y = 3^x \Rightarrow\right\} \frac{dy}{dx} = 3^x \ln 3$	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$	B1
	Either T: $y - 9 = 3^2 \ln 3(x-2)$ **or T:** $y = (3^2\ln 3)x + 9 - 18\ln 3$, where $9 = (3^2\ln 3)(2) + c$	See notes	M1
	{Cuts x-axis $\Rightarrow y = 0 \Rightarrow$}		
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2\ln 3)x + 9 - 18\ln 3$,	Sets $y = 0$ in their tangent equation and progresses to $x = ...$	M1
	So, $x = 2 - \frac{1}{\ln 3}$	$2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 **cso**
			[4]
(b)	$V = \pi\int\left(3^x\right)^2 \{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$	$V = \pi\int\left(3^x\right)^2$ with or without dx, which can be implied	B1 o.e.
	$= \{\pi\}\left(\frac{3^{2x}}{2\ln 3}\right)$ or $= \{\pi\}\left(\frac{9^x}{\ln 9}\right)$	Eg: either $3^{2x} \to \frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$, $\alpha \in$	M1
		$3^{2x} \to \frac{3^{2x}}{2\ln 3}$ or $9^x \to \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$	A1 o.e.

	$\left\{V = \pi\int_0^2 3^{2x}\,dx = \{\pi\}\left[\frac{3^{2x}}{2\ln 3}\right]_0^2\right\} = \{\pi\}\left(\frac{3^4}{2\ln 3} - \frac{1}{2\ln 3}\right)\ \left\{=\frac{40\pi}{\ln 3}\right\}$ — **Dependent on the previous method mark.** Substitutes $x = 2$ and $x = 0$ and subtracts the correct way round.	dM1
	$V_{cone} = \frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right)\ \left\{=\frac{27\pi}{\ln 3}\right\}$ — $V_{cone} = \frac{1}{3}\pi(9)^2(2 - \text{their }(a))$. See notes.	B1ft
	$\left\{\text{Vol}(S) = \frac{40\pi}{\ln 3} - \frac{27\pi}{\ln 3}\right\} = \underline{\frac{13\pi}{\ln 3}}$ — $\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$ etc., isw	A1 o.e.
	$\{\text{Eg: } p = 13\pi,\ q = \ln 3\}$	[6]
		10
(b)	**Alternative Method 1: Use of a substitution**	
	$V = \pi\int\left(3^x\right)^2\{dx\}$	B1 o.e.
	$\left\{u = 3^x \Rightarrow \frac{du}{dx} = 3^x\ln 3 = u\ln 3\right\}\ V = \{\pi\}\int\frac{u^2}{u\ln 3}\{du\} = \{\pi\}\int\frac{u}{\ln 3}\{du\}$	
	$= \{\pi\}\left(\frac{u^2}{2\ln 3}\right)$ — $\left(3^x\right)^2 \to \frac{u^2}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)u^2$, where $u = 3^x$	M1
	$\left(3^x\right)^2 \to \frac{u^2}{2(\ln 3)}$, where $u = 3^x$	A1
	$\left\{V = \pi\int_0^2\left(3^x\right)^2 dx = \{\pi\}\left[\frac{u^2}{2\ln 3}\right]_1^9\right\} = \{\pi\}\left(\frac{9^2}{2\ln 3} - \frac{1}{2\ln 3}\right)\ \left\{=\frac{40\pi}{\ln 3}\right\}$ — Substitutes limits of 9 and 1 in u (or 2 and 0 in x) and subtracts the correct way round.	dM1
	then apply the main scheme.	

	Question 8 Notes	
8. (a)	**B1**	$\frac{dy}{dx} = 3^x\ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$. Can be implied by later working.
	M1	Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{dy}{dx}$ which is a function of x or y to find m_T **and** • **either** applies $y - 9 = (\text{their } m_T)(x - 2)$, where m_T is a numerical value. • **or** applies $y = (\text{their } m_T)x + \text{their } c$, where m_T is a numerical value and c is found by solving $9 = (\text{their } m_T)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their ***tangent*** equation, where m_T is a numerical value, (seen or implied) and progresses to $x = ...$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda\ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda\ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda\ln 3}$ or $2 - \frac{\lambda}{\lambda\ln 3}$, where λ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their }\frac{dy}{dx}}$ or $\frac{1}{\text{their }\frac{dy}{dx}}$) is M0 M0 in part (a).
	Note	Candidates who invent a value for m_T (which bears no resemblance to their gradient function) **cannot** gain the 1st M1 and 2nd M1 mark in part (a).
	Note	A decimal answer of 1.089760773… (without a correct **exact** answer) is A0.

8. (b)	**B1**	A correct expression for the volume with or without dx
	Note	**Eg:** Allow B1 for $\pi\int\left(3^x\right)^2\{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$ or $\pi\int\left(e^{x\ln 3}\right)^2\{dx\}$ or $\pi\int\left(e^{2x\ln 3}\right)\{dx\}$ or $\pi\int e^{x\ln 9}\{dx\}$ with or without dx
	M1	Either $3^{2x}\to\frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x\to\frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$ $e^{2x\ln 3}\to\frac{e^{2x\ln 3}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)e^{2x\ln 3}$ or $e^{x\ln 9}\to\frac{e^{x\ln 9}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)e^{x\ln 9}$, etc where $\alpha \in$
	Note	$3^{2x}\to\frac{3^{2x+1}}{\pm\alpha(\ln 3)}$ or $9^x\to\frac{9^{x+1}}{\pm\alpha(\ln 3)}$ are allowed for M1
	Note	$3^{2x}\to\frac{3^{2x+1}}{2x+1}$ or $9^x\to\frac{9^{x+1}}{x+1}$ are both M0
	Note	M1 can be given for $9^{2x}\to\frac{9^{2x}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^{2x}$
	A1	Correct integration of 3^{2x}. Eg: $3^{2x}\to\frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x\to\frac{9^x}{\ln 9}$ or $e^{2x\ln 3}\to\frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$
	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x=2$ and $x=0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x=2$ and $x=0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	B1ft	$V_{\text{cone}}=\frac{1}{3}\pi(9)^2\left(2-\text{their answer to part }(a)\right)$. Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark.
	Note	Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark.
	A1	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$, etc. , where their answer is in the form $\frac{p}{q}$
	Note	The π in the volume formula is only needed for the 1st B1 mark and the final A1 mark.
	Note	A decimal answer of 37.17481128… (without a correct **exact** answer) is A0.
	Note	A candidate who applies $\int 3^x dx$ will either get B0 M0 A0 M0 **B0** A0 **or** B0 M0 A0 M0 **B1** A0
	Note	$\pi\int 3^{x^2}\,dx$ unless recovered is B0.
	Note	**Be careful! A correct answer may follow from incorrect working** $V=\pi\int_0^2 3^{x^2}\,dx-\frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right)=\pi\left[\frac{3^{x^2}}{2\ln 3}\right]_0^2-\frac{27\pi}{\ln 3}=\frac{\pi 3^4}{2\ln 3}-\frac{\pi}{2\ln 3}-\frac{27\pi}{\ln 3}=\frac{13\pi}{\ln 3}$ would score B0 M0 A0 dM0 M1 A0.

8. (b)	**2nd B1ft mark for finding the Volume of a Cone** $V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2\,\mathrm{d}x$ $= \pi\left[\frac{(9x\ln 3 - 18\ln 3 + 9)^3}{27\ln 3}\right]_{2-\frac{1}{\ln 3}\text{ or their part (a)answer}}^{2}$ **** $= \pi\left(\left(\frac{(18\ln 3 - 18\ln 3 + 9)^3}{27\ln 3}\right) - \left(\frac{\left(9\left(2-\frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$ $= \pi\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{(18\ln 3 - 9 - 18\ln 3 + 9)^3}{27\ln 3}\right)\right)$ $= \frac{27\pi}{\ln 3}$	Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.
8. (b)	**2nd B1ft mark for finding the Volume of a Cone** **Alternative method 2:** $V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2\,\mathrm{d}x$ $= \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(81x^2(\ln 3)^2 - 324x(\ln 3)^2 + 162x\ln 3 - 324\ln 3 + 324(\ln 3)^2 + 81\right)\mathrm{d}x$ $= \pi\left[27x^3(\ln 3)^2 - 162x^2(\ln 3)^2 + 81x^2\ln 3 - 324x\ln 3 + 324x(\ln 3)^2 + 81x\right]_{2-\frac{1}{\ln 3}}^{2}$ **** $= \pi\left(\begin{array}{l}\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right) \\ -\left(\begin{array}{l}27\left(2-\frac{1}{\ln 3}\right)^3(\ln 3)^2 - 162\left(2-\frac{1}{\ln 3}\right)^2(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)^2\ln 3 \\ -324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\end{array}\right)\end{array}\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l}27\left(8 - \frac{12}{\ln 3} + \frac{6}{(\ln 3)^2} - \frac{1}{(\ln 3)^3}\right)(\ln 3)^2 - 162\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)(\ln 3)^2 \\ + 81\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)\ln 3 - 324\left(2-\frac{1}{\ln 3}\right)\ln 3 \\ + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\end{array}\right)\right)$	Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l} 216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162 \\ +324\ln 3 - 324 + \frac{81}{\ln 3} - 648\ln 3 + 324 \\ + 648(\ln 3)^2 - 324\ln 3 + 162 - \frac{81}{\ln 3} \end{array}\right)\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

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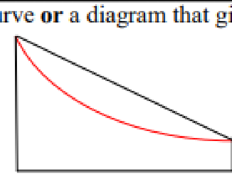
28.

Question Number	Scheme		Marks
3.	x: 1, 2, 3, 4; y: 1.42857, 0.90326, ***0.682116...***, 0.55556	$y = \dfrac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp)	0.68212	B1 **cao**
			[1]
(b)	$\frac{1}{2} \times 1 \times \underline{\left[1.42857 + 0.55556 + 2(0.90326 + \text{their } 0.68212)\right]}$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1 aef
		<u>For structure of</u> $\underline{[\ldots\ldots\ldots\ldots\ldots]}$	M1
	$\{= \frac{1}{2}(5.15489)\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	• Overestimate **and** a reason such as • {top of} <u>trapezia lie above the curve</u> • a diagram which gives reference to the extra area • concave or convex • $\dfrac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards		B1
			[1]
(d)	$\{u = \sqrt{x} \Rightarrow\}\ \dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{dx}{du} = 2u$		B1
	$\displaystyle\int \frac{10}{2u^2 + 5u} . 2u\, du$	Either $\left\{\displaystyle\int\right\} \dfrac{\pm ku}{\alpha u^2 \pm \beta u} \{du\}$ or $\left\{\displaystyle\int\right\} \dfrac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\}$	M1

	$\left\{= \int \frac{20}{2u+5}\,du\right\} = \frac{20}{2}\ln(2u+5)$	$\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right),\ \lambda \neq 0$ **with no other terms.**	M1
		$\frac{20}{2u+5} \to \frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$	A1 **cso**
	$\left\{\left[\frac{20}{2}\ln(2u+5)\right]_1^2\right\} = 10\ln(2(2)+5) - 10\ln(2(1)+5)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$		A1 oe **cso**
			[6] **11**

Question 3 Notes

3. (a)	**B1**	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or equivalent.
	M1	<u>For structure of trapezium rule</u>$\left[\ \ldots\ldots\ldots\ldots\ \right]$
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)
3. (b) contd	**Note**	Award B1M1A1 for $\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445$
		<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly
		award B1M0A0 for $\frac{1}{2}\times 1 + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556$ (nb: answer of 5.65489).
		award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)$ (nb: answer of 4.162825).
		<u>Alternative method: Adding individual trapezia</u>
		Area $\approx 1\times\left[\frac{1.42857+0.90326}{2} + \frac{0.90326+\text{"}0.68212\text{"}}{2} + \frac{\text{"}0.68212\text{"}+0.55556}{2}\right] = 2.577445$
	B1	**B1:** 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1:** anything that rounds to 2.5774
(c)	**B1**	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area
		eg. This diagram is sufficient. It must show the top of a trapezium lying above the curve. 
		or concave or convex **or** $\frac{d^2y}{dx^2} > 0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.

(d)	B1	$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $du = \frac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2u\,du$ or $\frac{dx}{du} = 2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$, $k, \alpha, \beta \neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**
	A1	**cso**. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent.
		So $\int\frac{10}{2u+5}\,du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

29.

Question Number	Scheme		Marks	
6. (i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int\frac{1}{4}e^{4x}\{dx\}$	$\pm\alpha xe^{4x} - \int\beta e^{4x}\{dx\}$, $\alpha \neq 0, \beta > 0$	M1	
		$\frac{1}{4}xe^{4x} - \int\frac{1}{4}e^{4x}\{dx\}$	A1	
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1	
				[3]
(ii)	$\int\frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1	
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1	
	$\{= -2(2x-1)^{-2}\{+c\}\}$	*{Ignore subsequent working}.*		[2]
(iii)	$\frac{dy}{dx} = e^x\operatorname{cosec}2y\operatorname{cosec}y$ $\quad y = \frac{\pi}{6}$ at $x = 0$			

	Main Scheme	
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y \sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	B1 oe
	$\int 2\sin y\cos y\sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ — Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
	Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = \mathrm{e}^x\ \{+c\}$ — $2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
	$\mathrm{e}^x \to \mathrm{e}^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = \mathrm{e}^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$ — Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$ — $\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$	A1
		[7]

	Alternative Method 1	
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y \sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ — $\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
	Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = \mathrm{e}^x\ \{+c\}$ — $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
	$\mathrm{e}^x \to \mathrm{e}^x$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = \mathrm{e}^0 + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$ — Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$ — $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	A1
		[7]
		12

	Question 6 Notes	
6. (i)	**M1**	Integration by parts is applied in the form $\pm\alpha x\mathrm{e}^{4x} - \int \beta\mathrm{e}^{4x}\{\mathrm{d}x\}$, where $\alpha \neq 0, \beta > 0$.
		(must be in this form).
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\{\mathrm{d}x\}$ or equivalent.
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}$ with/without $+c$. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	**M1**	$\pm\lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without $+c$. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.

(iii)	B1	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\text{cosec}\,2y\ \text{cosec}\,y} = \int e^x$ **or** $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\text{cosec}2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y \cos y$ **or** $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y,\ \mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y\ ,\ \alpha \neq 0, \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) **or** integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ **or** $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

$\int \sin 2y \sin y\ dy = \int e^x\ dx$		B1 oe
	Applies integration by parts **twice** to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	**M2**
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x\ \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]

30.

Question Number	Scheme		Marks
1. (a)	$\int x^2 e^x \, dx$, 1st Application: $\left\{ u = x^2 \Rightarrow \frac{du}{dx} = 2x ;\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \right\}$, 2nd Application: $\left\{ u = x \Rightarrow \frac{du}{dx} = 1 ;\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \right\}$		
	$= x^2 e^x - \int 2xe^x dx$	$x^2 e^x - \int \lambda x e^x \{dx\},\ \lambda > 0$	M1
		$x^2 e^x - \int 2xe^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)\ \{+c\}$	$\pm Ax^2e^x \pm Bxe^x \pm Ce^x$	M1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{ \left[x^2e^x - 2(xe^x - e^x) \right]_0^1 \right\}$ $= \left(1^2e^1 - 2(1e^1 - e^1)\right) - \left(0^2e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ **cso**	A1 oe
			[2]
			7

Notes for Question 1	
(a)	**M1:** Integration by parts is applied in the form $x^2e^x - \int \lambda xe^x \{dx\}$, where $\lambda > 0$. (must be in this form).
	A1: $x^2e^x - \int 2xe^x \{dx\}$ or equivalent.
	M1: **Either** achieving a result in the form $\pm Ax^2e^x \pm Bxe^x \pm C\int e^x \{dx\}$ (can be implied) (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$
	M1: $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$)
	A1: $x^2e^x - 2(xe^x - e^x)$ or $x^2e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+c$.
(b)	**M1:** **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1. So, just subtracting zero is M0.
	A1: $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$. **Note:** that 0.718... without seeing $e - 2$ or equivalent is A0. **WARNING:** Please note that this A1 mark is for correct solution only. So incorrect $[.......]_0^1$ leading to $e - 2$ is A0.
	Note: If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working.
	Note: 0.718... from no working is M0A0

31.

Question Number	Scheme		Marks
3. (a)	1.154701		B1 cao [1]
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{6}; \times\left[1+2(1.035276 + \text{their } 1.154701)+1.414214\right]$		B1; M1
	$= \frac{\pi}{12}\times 6.794168 = 1.778709023... = 1.7787$ (4 dp)	1.7787 or awrt 1.7787	A1 [3]
(c)	$V = \pi\int_0^{\frac{\pi}{2}}\left(\sec\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$	For $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$. Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	$\pm\lambda\tan\left(\frac{x}{2}\right)$	M1
		$2\tan\left(\frac{x}{2}\right)$ or equivalent	A1
	$= 2\pi$	2π	A1 cao cso [4] 8

Notes for Question 3	
(a)	**B1:** 1.154701 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times\frac{\pi}{6}$ or $\frac{\pi}{12}$ or awrt 0.262 **M1:** For structure of trapezium rule[............] **A1:** anything that rounds to 1.7787 **Note:** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 1.7787) **Note:** Working must be seen to demonstrate the use of the trapezium rule. **Note**: actual area is 1.762747174… Note: Award B1M1A1 for $\frac{\pi}{12}(1+1.414214)+\frac{\pi}{6}(1.035276 + \text{their } 1.154701) = 1.778709023...$ Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}+1+2(1.035276 + \text{their } 1.154701)+1.414214$ (nb: answer of 7.05596...). Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}$ $(1+1.414214)+2(1.035276 + \text{their } 1.154701)$ (nb: answer of 5.01199...). *Alternative method for part (b): Adding individual trapezia* Area $\approx \frac{\pi}{6}\times\left[\frac{1+1.035276}{2}+\frac{1.035276+1.154701}{2}+\frac{1.154701+1.414214}{2}\right] = 1.778709023...$ **B1:** $\frac{\pi}{6}$ and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 1.7787

Notes for Question 3 Continued	
3. (c)	**B1:** For a correct statement of $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$ or $\pi\int\sec^2\left(\frac{x}{2}\right)$ or $\pi\int\frac{1}{\left(\cos\left(\frac{x}{2}\right)\right)^2}\{dx\}$. Ignore limits and dx. Can be implied. Note: Unless a correct expression stated $\pi\int\sec\left(\frac{x^2}{4}\right)$ would be B0. **M1:** $\pm\lambda\tan\left(\frac{x}{2}\right)$ from any working. **A1:** $2\tan\left(\frac{x}{2}\right)$ or $\frac{1}{\left(\frac{1}{2}\right)}\tan\left(\frac{x}{2}\right)$ from any working. **A1:** 2π **from a correct solution only.** **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 6.283... without correct exact answer is A0. **Note:** The B1 mark can be implied by later working – as long as it is clear that the candidate has applied $\pi\int y^2$ in their working. **Note:** Writing the correct formula of $V=\pi\int y^2\{dx\}$, but incorrectly applying it is B0.

32.

Question Number	Scheme		Marks
5. (a)	$\{x=u^2\Rightarrow\}\ \frac{dx}{du}=2u$ or $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$		B1
	$\left\{\int\frac{1}{x(2\sqrt{x}-1)}dx\right\}=\int\frac{1}{u^2(2u-1)}2u\,du$		M1
	$=\int\frac{2}{u(2u-1)}du$		A1 * cso
			[3]
(b)	$\frac{2}{u(2u-1)}\equiv\frac{A}{u}+\frac{B}{(2u-1)}\Rightarrow 2\equiv A(2u-1)+Bu$ $u=0\Rightarrow 2=-A\Rightarrow A=-2$ $u=\frac{1}{2}\Rightarrow 2=\frac{1}{2}B\Rightarrow B=4$	See notes	M1 A1
	So $\int\frac{2}{u(2u-1)}du=\int\frac{-2}{u}+\frac{4}{(2u-1)}du$	Integrates $\frac{M}{u}+\frac{N}{(2u-1)}$, $M\neq 0$, $N\neq 0$ to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$	M1
	$=-2\ln u+2\ln(2u-1)$	At least one term correctly followed through	A1 ft
		$-2\ln u+2\ln(2u-1)$.	A1 cao
	So, $\left[-2\ln u+2\ln(2u-1)\right]_1^3$		
	$=\left(-2\ln 3+2\ln(2(3)-1)\right)-\left(-2\ln 1+2\ln(2(1)-1)\right)$	Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$=-2\ln 3+2\ln 5-(0)$		
	$=2\ln\left(\frac{5}{3}\right)$	$2\ln\left(\frac{5}{3}\right)$	A1 cso cao
			[7] **10**

Notes for Question 5	
(a)	**B1:** $\frac{\mathrm{d}x}{\mathrm{d}u} = 2u$ or $\mathrm{d}x = 2u\,\mathrm{d}u$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ or $\mathrm{d}u = \frac{\mathrm{d}x}{2\sqrt{x}}$ **M1:** A full substitution producing an integral in u only (including the $\mathrm{d}u$) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "$\mathrm{d}x$" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1). **A1*:** leading to the result printed on the question paper (including the $\mathrm{d}u$). (Integral sign is needed).
(b)	**M1:** Writing $\frac{2}{u(2u-1)} \equiv \frac{A}{u} + \frac{B}{(2u-1)}$ or writing $\frac{1}{u(2u-1)} \equiv \frac{P}{u} + \frac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q). **A1:** Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign). **M1:** Integrates $\frac{M}{u} + \frac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$ or $\pm\mu\ln\left(u-\frac{1}{2}\right)$ **A1ft:** At least one term correctly followed through from their A or from their B (or their P and their Q). **A1:** $-2\ln u + 2\ln(2u-1)$
Notes for Question 5 Continued	
5. (b) ctd	**M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the correct way round. **Note:** If a candidate just writes $(-2\ln 3 + 2\ln(2(3)-1))$ oe, this is ok for M1. **A1:** $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a = 5, b = 3$). **Important note: Award M0A0M1A1A0** for a candidate who writes $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **AS EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.** **Important note: Award M0A0M0A0A0** for a candidate who writes down either $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1)$ **or** $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Important note: Award M1A1M1A1A1** for a candidate who writes down $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Note:** In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of M1A0 M1A1ftA0 M1A0.

33.

Question Number	Scheme		Marks
6.	$\frac{d\theta}{dt} = \lambda(120-\theta), \quad \theta \leqslant 100$		
(a)	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$ or $\int \frac{1}{\lambda(120-\theta)}\,d\theta = \int dt$		B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$	See notes	M1 A1; M1 A1
	$\{t=0, \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either...	***or...***	
	$-\lambda t = \ln(120-\theta) - \ln 100$ $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$ $e^{-\lambda t} = \frac{120-\theta}{100}$	$\lambda t = \ln 100 - \ln(120-\theta)$ $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$ $e^{\lambda t} = \frac{100}{120-\theta}$	dddM1
	$100e^{-\lambda t} = 120-\theta$	$(120-\theta)e^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100e^{-\lambda t}$	
	leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01, \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$ $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379... = 161$ (s) (nearest second)	awrt 161	A1
			[3] **11**

Notes for Question 6

(a)

B1: Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

Either

M1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$

A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$

M1: $\int \lambda\,\mathrm{d}t \to \lambda t$

A1: $\int \lambda\,\mathrm{d}t \to \lambda t + c$

or

$\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant.

$\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$,

$\int 1\,\mathrm{d}t \to t$

or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation.

IMPORTANT: $+c$ can be on either side of their equation for the 2nd A1 mark.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated or changed equation containing c (or A or $\ln A$).

Note that this mark can be implied by the correct value of c. { Note that $-\ln 100 = -4.60517...$ }.

dddM1: Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded.

A1*: This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either:

(1): $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100\mathrm{e}^{-\lambda t} = 120-\theta \Rightarrow \theta = 120 - 100\mathrm{e}^{-\lambda t}$

or (2): $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)\mathrm{e}^{\lambda t} = 100 \Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t} \Rightarrow \theta = 120 - 100\mathrm{e}^{-\lambda t}$

is required for A1.

Note: $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).

(b)

M1: Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working.

dM1: Candidate uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to $t = ...$

Note: that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b).

A1: awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).

***Aliter* 6. (a) Way 2**	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$	B1
	$-\ln(120-\theta) = \lambda t + c$ See notes	M1 A1; M1 A1
	$-\ln(120-\theta) = \lambda t + c$	
	$\ln(120-\theta) = -\lambda t + c$	
	$120-\theta = A\mathrm{e}^{-\lambda t}$	
	$\theta = 120 - A\mathrm{e}^{-\lambda t}$	
	$\{t = 0, \theta = 20 \Rightarrow\}\ 20 = 120 - A\mathrm{e}^{0}$	M1
	$A = 120 - 20 = 100$	
	So, $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	dddM1 A1 *
		[8]

Notes for Question 6 Continued	
(a)	**B1M1A1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **dddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on all three previous method marks being awarded. **Note:** $\ln(120-\theta) = -\lambda t + c$ leading to $120-\theta = e^{-\lambda t} + e^{c}$ or $120-\theta = e^{-\lambda t} + A$, would be dddM0. **A1*:** Same as the original scheme. **Note:** The jump from $\ln(120-\theta) = -\lambda t + c$ to $120-\theta = Ae^{-\lambda t}$ ***with no incorrect working*** is condoned in part (a).

Aliter 6. (a) Way 3	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,d\theta = \int \lambda\,dt\right\}$		B1
	$-\ln\lvert\theta-120\rvert = \lambda t + c$	*Modulus required for 1st A1.*	M1 **A1**
			M1 A1
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0) + c$	*Modulus not required here!*	M1
	$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$		

then either...	***or...***
$-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$	$\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$
$-\lambda t = \ln\left\lvert\frac{\theta-120}{100}\right\rvert$	$\lambda t = \ln\left\lvert\frac{100}{\theta-120}\right\rvert$
As $\theta \leqslant 100$	
$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$	$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$
$e^{-\lambda t} = \frac{120-\theta}{100}$	$e^{\lambda t} = \frac{100}{120-\theta}$
$100e^{-\lambda t} = 120-\theta$	$(120-\theta)e^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100e^{-\lambda t}$
leading to $\theta = 120 - 100e^{-\lambda t}$	

Understanding of modulus is required here! **dddM1**

A1 *

[8]

	B1: Mark as in the original scheme. **M1:** Mark as in the original scheme ignoring the modulus. **A1:** $\int \frac{1}{120-\theta}\,d\theta \rightarrow -\ln\lvert\theta-120\rvert$. ***(The modulus is required here).*** **M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus. **dddM1:** Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. **Note:** This mark is dependent on all three previous method marks being awarded. **A1:** Mark as in the original scheme.

Notes for Question 6 Continued		
Aliter 6. (a) Way 4	***Use of an integrating factor***	
	$\frac{d\theta}{dt} = \lambda(120-\theta) \Rightarrow \frac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\frac{d}{dt}\left(e^{\lambda t}\theta\right) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t=0, \theta = 20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1

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34.

Question Number	Scheme		
2. (a)	$\int \frac{1}{x^3}\ln x \, dx$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array}\right\}$		
		In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$	M1
	$= \underline{\frac{-1}{2x^2}\ln x} - \underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x}\,dx}}$	$\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	<u>A1</u>
		$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	<u>A1</u>
	$\left\{= \frac{-1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}\,dx\right\}$		
	$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$	$\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}$.	dM1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\right]_1^2\right\} = \left(-\frac{1}{2(2)^2}\ln 2 - \frac{1}{4(2)^2}\right) - \left(-\frac{1}{2(1)^2}\ln 1 - \frac{1}{4(1)^2}\right)$	Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$= \frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$, etc, or awrt 0.1	or equivalent.	A1
			[2] **7**

(a)	**M1**: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$ or equivalent.
	A1: $\frac{-1}{2x^2}\ln x$ simplified or un-simplified.
	A1: $-\int \frac{-1}{2x^2}.\frac{1}{x}$ or equivalent. You can ignore the $\mathrm{d}x$.
	dM1: Depends on the previous M1. $\pm\int \mu\frac{1}{x^2}.\frac{1}{x} \rightarrow \pm\beta x^{-2}$.
	A1: $-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ or $= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\{+c\}$ or $\frac{x^{-2}}{-2}\ln x - \frac{x^{-2}}{4}\{+c\}$ or $\frac{-1-2\ln x}{4x^2}\{+c\}$ or equivalent. You can ignore subsequent working after a correct stated answer.
(b)	**M1:** Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round.
	A1: ***Two term exact answer*** of either $\frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right)+3}{16}$ or $0.1875 - 0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined.
	Note: Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.

2. (b) ctd

Note: Decimal answer is 0.100856... in part (b).

Special Case (b) M1A1: for a candidate who finds an answer in (a) which is out by a factor of -1.

Award **SC M1A1** for $\frac{1}{2x^2}\ln x + \frac{1}{2}\left(\frac{1}{2x^2}\right)\{+c\}$ in (a) leading to $-\frac{3}{16} + \frac{1}{8}\ln 2$, etc or awrt -0.1 in (b).

Alternative Solution

$$\int \frac{1}{x^3}\ln x\,\mathrm{d}x, \quad \left\{\begin{array}{ll} u = x^{-3} & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -3x^{-4} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x & \Rightarrow v = x\ln x - x \end{array}\right\}$$

$$\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int (x\ln x - x)\frac{-3}{x^4}\mathrm{d}x$$

Working	Notes	Marks
	$k\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) \pm \int \frac{\lambda}{x^3}\mathrm{d}x$ where $k \neq 1$	M1
$-2\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$	Any one of $\frac{1}{x^3}(x\ln x - x)$ or $-\int \frac{3}{x^3}\mathrm{d}x$	A1
	$\frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\mathrm{d}x$ and $k = -2$	A1
$-2\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) + \frac{3}{2x^2}\ \{+c\}$	$\pm\int \mu\frac{1}{x^3} \rightarrow \pm\beta x^{-2}$.	dM1
$\int \frac{1}{x^3}\ln x\,\mathrm{d}x = -\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}\ \{+c\}$ $= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\{+c\}$	$-\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}$ or equivalent with/without $+c$.	A1

35.

Question Number	Scheme		Marks
4. (a)	1.0981		B1 **cao** [1]
(b)	Area $\approx \frac{1}{2}\times 1 ; \times \underline{\left[0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333\right]}$		B1; <u>M1</u>
	$= \frac{1}{2}\times 5.6863 \;=\; 2.84315 \;= 2.843 \text{ (3 dp)}$	2.843 or awrt 2.843	A1 [3]
(c)	$\{u = 1+\sqrt{x}\} \Rightarrow \underline{\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{dx}{du} = 2(u-1)}$		<u>B1</u>
	$\left\{\int \frac{x}{1+\sqrt{x}}\,dx = \right\} \int \frac{(u-1)^2}{u}.\,2(u-1)\;du$	$\int \frac{(u-1)^2}{u}......$	M1
		$\int \frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$= 2\int \frac{(u-1)^3}{u}\,du \;= \{2\}\int \frac{(u^3 - 3u^2 + 3u - 1)}{u}\,du$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$= \{2\}\int \left(u^2 - 3u + 3 - \frac{1}{u}\right)du$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$= \{2\}\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	$\int \frac{(u-1)^3}{u} \to \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$		
	$= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1
			[8] **12**

(a)	**B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$
	M1: <u>For structure of trapezium rule</u>[<u>............</u>]
	A1: anything that rounds to 2.843
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>Note</u>: actual area is 2.85573645…
	<u>Note:</u> Award B1M1 A1 for $\frac{1}{2}(0.5 + 1.3333) + (0.8284 + \text{their } 1.0981) = 2.84315$
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**
	Award B1M0A0 for $\frac{1}{2}\times 1 + 0.5 + 2(0.8284 + \text{their } 1.0981) + 1.3333$ (nb: answer of 6.1863).
	Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5 + 1.3333) + 2(0.8284 + \text{their } 1.0981)$ (nb: answer of 4.76965).

4. (b) ctd

Alternative method for part (b): Adding individual trapezia

$$\text{Area} \approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

Note: This question appears as B1 M1 **B1** M1 M1 A1 M1 A1 on ePEN,
but is now marked as B1 M1 **A1** M1 M1 A1 M1 A1.

B1: $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $du = \frac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2(u-1)du$ or $\frac{dx}{du} = 2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1: $\frac{x}{1+\sqrt{x}}\,dx$ becoming $\frac{(u-1)^2}{u}.\,2(u-1)\{du\}$ or $\frac{(u-1)^2}{u}.\frac{2}{(u-1)^{-1}}\{du\}$.
You can ignore the integral sign and the du.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3 \pm Bu^2 \pm Cu \pm D$
where $A \neq 0$, $B \neq 0$, $C \neq 0$ and $D \neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in ***their cubic*** by u.
Ie. $\frac{(u^3-3u^2+3u-1)}{u} \to u^2 - 3u + 3 - \frac{1}{u}$

2nd A1: $\int \frac{(u-1)^3}{u}\,du \to \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.
Note: $\left(\frac{4^3}{3} - \frac{3(4)^2}{2} + 3(4) - \ln 4\right) - \left(\frac{1^3}{3} - \frac{3(1)^2}{2} + 3(1) - \ln 1\right)$ is M0.

You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does explicitly give **some evidence**.
Note: For correct integral and limits decimals gives: $(6.802775...) - (3.947038...) = 2.85573...$

3rd A1: Exact answer of $\frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6} + \ln 2 - \ln 3\right)$
or $\frac{22}{6} + 2\ln\left(\frac{2}{3}\right)$, etc. **Note:** that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

Alternative method for 2nd M1 and 3rd M1 mark

$$\{2\}\int \frac{(u-1)^2}{u}.(u-1)\,du = \{2\}\int \frac{(u^2-2u+1)}{u}.(u-1)\,du$$

$$= \{2\}\int\left(u - 2 + \frac{1}{u}\right).(u-1)\,du = \{2\}\int\left(u^2 - ...\right)du$$

An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$. — **2nd M1**

$$= \{2\}\int\left(u^2 - 2u + 1 - u + 2 - \frac{1}{u}\right)du$$

to give three out of four of $\pm Au^2, \pm Bu, \pm C$ or $\pm\frac{D}{u}$ — **3rd M1**

$$= \{2\}\int\left(u^2 - 3u + 3 - \frac{1}{u}\right)du$$

4. (c) ctd	**Final two marks in part (c):** $u = 1 + \sqrt{x}$	
	$\text{Area}(R) = \left[\frac{2(1+\sqrt{x})^3}{3} - 3(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 2\ln(1+\sqrt{x})\right]_1^4$	
	$= \left(\frac{2(1+\sqrt{4})^3}{3} - 3(1+\sqrt{4})^2 + 6(1+\sqrt{4}) - 2\ln(1+\sqrt{4})\right)$ $- \left(\frac{2(1+\sqrt{1})^3}{3} - 3(1+\sqrt{1})^2 + 6(1+\sqrt{1}) - 2\ln(1+\sqrt{1})\right)$	**M1:** Applies limits of 4 and 1 in x and subtracts either way round.
	$= (18 - 27 + 18 - 2\ln 3) - \left(\frac{16}{3} - 12 + 12 - 2\ln 2\right)$	
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc	**A1:** Correct exact answer or equivalent.
	Alternative method for the final 5 marks in part (b)	
	$\int \frac{(u-1)^3}{u}\,du$, $\left\{\begin{array}{l} "u" = u^{-1} \Rightarrow \frac{d"u"}{dx} = -u^{-2} \\ \frac{dv}{dx} = (u-1)^3 \Rightarrow v = \frac{(u-1)^4}{4} \end{array}\right\}$	
	$= \frac{(u-1)^4}{4u} - -\frac{1}{4}\int \frac{(u-1)^4}{u^2}\,du$	
	$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2}\,du$	**M1:** Applies integration by parts and expands to give a five term quartic.
	$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2}\,du$	**M1:** Dividing at least 4 terms.
	$= \frac{(u-1)^4}{4u} + \frac{1}{4}\left(\frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u}\right)$	**A1:** Correct Integration.
	$\int_2^3 \frac{(u-1)^3}{u}\,du = \left[\frac{(u-1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u}\right]_2^3$	
	$= \left(\frac{16}{12} + \frac{27}{12} - \frac{9}{2} + \frac{9}{2} - \ln 3 - \frac{1}{12}\right) - \left(\frac{1}{8} + \frac{8}{12} - \frac{4}{2} + \frac{6}{2} - \ln 2 - \frac{1}{8}\right)$	**M1**
	$= (7 - \ln 3) - \left(\frac{5}{3} - \ln 2\right)$	
	$= \frac{11}{6} + \ln\frac{2}{3}$	
	$\text{Area}(R) = 2\int_2^3 \frac{(u-1)^3}{u}\,du = 2\left(\frac{11}{6} + \ln\frac{2}{3}\right)$	**A1**

36.

Question Number	Scheme		Marks
6. (a)	$\{y=0 \Rightarrow\}\ 1-2\cos x = 0$	$1-2\cos x = 0$, seen or implied.	M1
	$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	At least one correct value of x. (See notes).	A1
		Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$	A1 **cso**
			[3]
(b)	$V = \pi\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1-2\cos x)^2\ dx$	For $\pi\int(1-2\cos x)^2$. Ignore limits and dx	B1
	$\left\{\int (1-2\cos x)^2\ dx\right\} = \int (1-4\cos x + 4\cos^2 x)dx$		
	$= \int 1 - 4\cos x + 4\left(\frac{1+\cos 2x}{2}\right) dx$	$\cos 2x = 2\cos^2 x - 1$ See notes.	M1
	$= \int (3 - 4\cos x + 2\cos 2x)\ dx$		
	$= 3x - 4\sin x + \frac{2\sin 2x}{2}$	Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$.	M1
		Correct integration.	A1
	$V = \{\pi\}\left(\left(3(\frac{5\pi}{3}) - 4\sin(\frac{5\pi}{3}) + \frac{2\sin(\frac{10\pi}{3})}{2}\right) - \left(3(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3}) + \frac{2\sin(\frac{2\pi}{3})}{2}\right)\right)$	Applying limits the correct way round. Ignore π.	ddM1
	$= \pi\left(\left(5\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}\right) - \left(\pi - 2\sqrt{3} + \frac{\sqrt{3}}{2}\right)\right)$		
	$= \pi((18.3060...) - (0.5435...)) = 17.7625\pi = 55.80$		
	$= \pi(4\pi + 3\sqrt{3})$ or $4\pi^2 + 3\pi\sqrt{3}$	Two term exact answer.	A1
			[6] **9**

6. (a)	**M1:** $1-2\cos x = 0$.
	This can be implied by either $\cos x = \frac{1}{2}$ or any one of the correct values for x in radians or in degrees.
	1st A1: Any one of either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24 .
	2nd A1: Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
(b)	**Note:** This part appears as M1 M1 M1 A1 M1 A1 on ePEN, but is now marked as **B1** M1 M1 A1 M1 A1.
	B1: For $\pi\int(1-2\cos x)^2$. Ignore limits and dx.
	1st M1: Any correct form of $\cos 2x = 2\cos^2 x - 1$ used or written down in the same variable.
	This can be implied by $\cos^2 x = \frac{1+\cos 2x}{2}$ or $4\cos^2 x \to 2 + 2\cos 2x$ or $\cos 2A = 2\cos^2 A - 1$.
	2nd M1: Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$.
	Do not worry about the signs when integrating $\cos x$ or $\cos 2x$ for this mark.
	Note: $\int (1-2\cos x)^2 = \int 1 + 4\cos^2 x$ is ok for an attempt at $\int y^2$.

	1st A1: Correct integration. Eg. $3x - 4\sin x + \frac{2\sin 2x}{2}$ or $x - 4\sin x + \frac{2\sin 2x}{2} + 2x$ oe. **3rd ddM1:** Depends on both of the two previous method marks. (Ignore π). Some evidence of substituting their $x = \frac{5\pi}{3}$ and their $x = \frac{\pi}{3}$ and subtracting the correct way round. You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give **some evidence.** **Note:** For correct integral and limits decimals gives: $\pi((18.3060...) - (0.5435...)) = 17.7625\pi = 55.80$ **2nd A1:** ***Two term*** exact answer of either $\pi\left(4\pi + 3\sqrt{3}\right)$ or $4\pi^2 + 3\pi\sqrt{3}$ or equivalent. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 58.802... without correct exact answer is A0. **Note:** Applying $\int(1 - 2\cos x)\,dx$ will usually be given no marks in this part.

37.

Question Number	Scheme		Marks
8. (a)	$\left\{\frac{d\theta}{dt} = \frac{(3-\theta)}{125}\right\} \Rightarrow \int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt$ or $\int \frac{125}{3-\theta}\,d\theta = \int dt$		B1
	$-\ln(\theta - 3) = \frac{1}{125}t\ \{+c\}$ or $-\ln(3-\theta) = \frac{1}{125}t\ \{+c\}$	See notes.	M1 A1
	$\ln(\theta - 3) = -\frac{1}{125}t + c$		
	$\theta - 3 = e^{-\frac{1}{125}t + c}$ or $e^{-\frac{1}{125}t}e^{c}$ $\theta = Ae^{-0.008t} + 3$ *	***Correct*** completion to $\theta = Ae^{-0.008t} + 3$.	A1
			[4]
(b)	$\{t = 0,\ \theta = 16 \Rightarrow\}$ $16 = Ae^{-0.008(0)} + 3;\ \Rightarrow \underline{A = 13}$	***See notes.***	M1; A1
	$10 = 13e^{-0.008t} + 3$	Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. ***See notes.***	M1
	$e^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right)$	Correct algebra to $-0.008t = \ln k$, where k is a positive value. **See *notes.***	M1
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	awrt 77	A1
			[5] **9**

8. (a)	**Note:** This part appears as M1 M1 A1 A1 on ePEN, but is now marked as B1 M1 A1 A1. **B1:** Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1:** **Both** $\pm\lambda\ln(3-\theta)$ or $\pm\lambda\ln(\theta-3)$ **and** $\pm\,\mu t$ where λ and μ are constants. **A1:** For $-\ln(\theta-3)=\frac{1}{125}t$ or $-\ln(3-\theta)=\frac{1}{125}t$ or $-125\ln(\theta-3)=t$ or $-125\ln(3-\theta)=t$ **Note:** $+c$ is not needed for this mark. **A1:** Correct completion to $\theta=Ae^{-0.008t}+3$. **Note:** $+c$ is needed for this mark. **Note:** $\ln(\theta-3)=-\frac{1}{125}t+c$ leading to $\theta-3=e^{-\frac{1}{125}t}+e^{c}$ or $\theta-3=e^{-\frac{1}{125}t}+A$, would be final A0. **Note:** From $-\ln(\theta-3)=\frac{1}{125}t+c$, then $\ln(\theta-3)=-\frac{1}{125}t+c$ $\Rightarrow \theta-3=e^{-\frac{1}{125}t+c}$ **or** $\theta-3=e^{-\frac{1}{125}t}e^{c}$ $\Rightarrow \theta=Ae^{-0.008t}+3$ is required for A1. **Note:** From $-\ln(3-\theta)=\frac{1}{125}t+c$, then $\ln(3-\theta)=-\frac{1}{125}t+c$ $\Rightarrow 3-\theta=e^{-\frac{1}{125}t+c}$ **or** $3-\theta=e^{-\frac{1}{125}t}e^{c}$ $\Rightarrow \theta=Ae^{-0.008t}+3$ is sufficient for A1. **Note:** The jump from $3-\theta=Ae^{-\frac{1}{125}t}$ to $\theta=Ae^{-0.008t}+3$ is fine. **Note:** $\ln(\theta-3)=-\frac{1}{125}t+c \Rightarrow \theta-3=Ae^{-\frac{1}{125}t}$, where candidate writes $A=e^{c}$ is also acceptable.

8. (b)	**Note:** This part appears as **B1 M1** M1 M1 A1 on ePEN, but is now marked as **M1 A1** M1 M1 A1. **Note:** You can recover work for part (b) in part (a). **M1:** Substitutes $\theta=16,\,t=0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark. **A1:** $A=13$. **Note:** $\theta=13e^{-0.008t}+3$ without any working implies the first two marks, M1A1. **M1:** Substitutes $\theta=10$ into an equation **of the form** $\theta=Ae^{-0.008t}+3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1. **M1:** Uses correct algebra to rearrange **their equation** into the form $-0.008t=\ln k$, where k is ***a positive numerical value***. **A1:** awrt 77 or awrt 1 hour 17 minutes. ***<u>Alternative Method 1 for part (b)</u>*** $\int\frac{1}{3-\theta}\,\mathrm{d}\theta=\int\frac{1}{125}\,\mathrm{d}t \Rightarrow -\ln(\theta-3)=\frac{1}{125}t+c$ $\{t=0\,,\,\theta=16\Rightarrow\}$ $-\ln(16-3)=\frac{1}{125}(0)+c$ $\Rightarrow c=-\ln 13$ $-\ln(\theta-3)=\frac{1}{125}t-\ln 13$ or $\ln(\theta-3)=-\frac{1}{125}t+\ln 13$ $-\ln(10-3)=\frac{1}{125}t-\ln 13$ $\ln 13-\ln 7=\frac{1}{125}t$ $t=77.3799...=77$ (nearest minute)	**M1:** Substitutes $t=0,\theta=16$, into $-\ln(\theta-3)=\frac{1}{125}t+c$ **A1:** $c=-\ln 13$ **M1:** Substitutes $\theta=10$ into an equation **of the form** $\pm\lambda\ln(\theta-3)=\pm\frac{1}{125}t\pm\mu$ where λ,μ are numerical values. **M1:** Uses correct algebra to rearrange **their equation** into the form $\pm0.008t=\ln C-\ln D$, where C, D are ***positive numerical values***. **A1:** awrt 77.

	Alternative Method 2 for part (b)	
	$\int \frac{1}{3-\theta}\,d\theta = \int \frac{1}{125}\,dt \Rightarrow -\ln\lvert 3-\theta\rvert = \frac{1}{125}t + c$	
	$\{t=0,\ \theta=16 \Rightarrow\}\ -\ln\lvert 3-16\rvert = \frac{1}{125}(0) + c$	**M1:** Substitutes $t=0, \theta=16$, into $-\ln(3-\theta) = \frac{1}{125}t + c$
	$\Rightarrow c = -\ln 13$	**A1:** $c = -\ln 13$
	$-\ln\lvert 3-\theta\rvert = \frac{1}{125}t - \ln 13$ or $\ln\lvert 3-\theta\rvert = -\frac{1}{125}t + \ln 13$	
	$-\ln(3-10) = \frac{1}{125}t - \ln 13$	**M1:** Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3-\theta) = \pm\frac{1}{125}t \pm\mu$ where λ, μ are numerical values.
	$\ln 13 - \ln 7 = \frac{1}{125}t$	**M1:** Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.
	$t = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77.

8. (b)	***Alternative Method 3 for part (b)***	
	$\int_{16}^{10} \frac{1}{3-\theta}\,d\theta = \int_0^t \frac{1}{125}\,dt$	
	$= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_0^t$	
		M1A1: $\ln 13$
	$-\ln 7 - -\ln 13 = \frac{1}{125}t$	**M1:** Substitutes limit of $\theta = 10$ correctly. **M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.
	$t = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77.
	Please escalate responses to review for candidates achieving 77 where you are not convinced of the method or if 77 is achieved and there are errors in working.	

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38.

Question Number	Scheme	Marks
1.	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $\quad (1 = A)$	M1
	$x \to \frac{1}{3}$ $\quad 1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2 $\quad 0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

39.

Question Number	Scheme		Marks	
4.	$\int y \, dy = \int \frac{3}{\cos^2 x} dx$	Can be implied. Ignore integral signs	B1	
	$= \int 3\sec^2 x \, dx$			
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2, x = \frac{\pi}{4}$			
	$\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to			
	$C = -1$			
	$\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	**(5)**
				[5]

40.

Question Number	Scheme		Marks	
7.	(a)		M1	

x	1	2	3	4
y	ln2	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	2ln8
	0.6931	1.9605	3.1034	4.1589

Question Number	Scheme		Marks	
	Area $= \frac{1}{2} \times 1(\ ...)$		B1	
	$\approx \ ... \ (0.6931 + 2(1.9605 + 3.1034) + 4.1589)$		M1	
	$\approx \frac{1}{2} \times 14.97989 \ ... \ \approx 7.49$	7.49 cao	A1	**(4)**
	(b) $\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{3}{2}} \times \frac{1}{x} dx$		M1 A1	
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{1}{2}} dx$			
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}} \quad (+C)$		M1 A1	**(4)**

	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}}\ln 8 - \frac{4}{9}4^{\frac{3}{2}}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$	M1	
	$= (16\ln 2 - \ ...) - \ ...$ Using or implying $\ln 2^n = n\ln 2$	M1	
	$= \frac{46}{3}\ln 2 - \frac{28}{9}$	A1	(3)
			[11]

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41.

Question Number	Scheme	Marks
2. (a)	$\int x\sin 3x \, dx = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \{dx\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x \, dx = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \{dx\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \ \{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \ \{+c\}\right\}$ *Ignore subsequent working*	[3]
		6

(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \rightarrow u' = 1$ and $v' = \sin 3x \rightarrow v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without $+ c$. Can be un-simplified.
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \rightarrow u' = 2x$ or x and $v' = \cos 3x \rightarrow v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without $+ c$, can be un-simplified. You can ignore subsequent working here. **Special Case**: If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.

42.

Question Number	Scheme		Marks
4.	Volume $= \pi\int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 dx$	Use of $V = \pi\int y^2\,dx$.	B1
	$= (\pi)\left[\frac{1}{3}\ln(3x^2+4)\right]_0^2$	$\pm k\ln(3x^2+4)$	M1
		$\frac{1}{3}\ln(3x^2+4)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right) - \left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \frac{1}{3}\pi\ln 4$	$\frac{1}{3}\pi\ln 4$ or $\frac{2}{3}\pi\ln 2$	A1 oe isw
			[5] **5**

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and dx. This can be implied by later working,

but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note:** $\pi\left(\int y\right)^2$ would be B0.

Working in x

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16-\ln 4)$ would be A0.

Working in u: where $u=3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln(pu)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

A1: As above!

43.

Question Number	Scheme		Marks
6. (a)	0.73508		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}$; $\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 <u>M1</u>
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp)	awrt 1.1504	A1 **[3]**
(c)	$\{u = 1+\cos x\} \Rightarrow \underline{\frac{du}{dx} = -\sin x}$		<u>B1</u>
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,dx = \right\} \int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,dx$	$\sin 2x = 2\sin x\cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\,du \quad \left\{= 4\int \frac{(1-u)}{u}\,du\right\}$		M1
	$= 4\int\left(\frac{1}{u}-1\right)du = 4(\ln u - u) + c$		dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x) + c \;=\; 4\ln(1+\cos x) - 4\cos x + k$	**AG**	A1 **cso** **[5]**
(d)	$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$	Applying limits $x = \frac{\pi}{2}$ and $x = 0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$		
	$= 4 - 4\ln 2 \;\{= 1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \lvert(4-4\ln 2) - 1.1504...\rvert$ $= 0.0770112776... = 0.077$ (2sf)	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
			12

(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196 **M1:** <u>For structure of trapezium rule</u>$\left[\underline{\text{............}}\right]$; (0 can be implied). **A1:** anything that rounds to 1.1504 <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8} + 2(\text{their } 0.73508+1.17157+1.02280)$ (nb: answer of 6.0552). Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}(0+0)+2(\text{their } 0.73508+1.17157+1.02280)$ (nb: answer of 5.8589). <u>*Alternative method for part (b): Adding individual trapezia*</u> Area $\approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$ B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504

6. (c)	**B1:** $\frac{du}{dx} = -\sin x$ or $du = -\sin x\,dx$ or $\frac{dx}{du} = \frac{1}{-\sin x}$ oe. **B1:** For seeing, applying or implying $\sin 2x = 2\sin x\cos x$. **M1:** After applying substitution candidate achieves $\pm k\int\frac{(u-1)}{u}(du)$ or $\pm k\int\frac{(1-u)}{u}(du)$. Allow M1 for "invisible" brackets here, eg: $\pm\int\frac{(\lambda u-1)}{u}(du)$ or $\pm\int\frac{(-\lambda+u)}{u}(du)$, where λ is a positive constant. **dM1:** An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)du \to \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded. **Alternative method:** Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below). **A1:** Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$ As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \to 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \to 4\ln(1+\cos x) - 4\cos x + k$ **Note**: that this mark is also for a correct solution only. **Note**: those candidates who attempt to find the value of k will usually achieve A0.
(d)	**M1:** Substitutes limits of $x = \frac{\pi}{2}$ and $x = 0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note** that: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0. **A1:** $4(1-\ln 2)$ or $4-4\ln 2$ or awrt 1.2, however found. This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3 **A1:** For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0. *Alternative method for dM1 in part (c)* $\int\frac{(1-u)}{u}du = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int\frac{u}{u}du\right) = ((1-u)\ln u + u\ln u - u)$ or $\int\frac{(u-1)}{u}du = \left((u-1)\ln u - \int\ln u\,du\right) = \left((u-1)\ln u - \left(u\ln u - \int\frac{u}{u}du\right)\right) = ((u-1)\ln u - u\ln u + u)$ So **dM1** is for $\int\frac{(1-u)}{u}du$ going to $((1-u)\ln u + u\ln u - u)$ or $((u-1)\ln u - u\ln u + u)$ oe. *Alternative method for part (d)* **M1A1** for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)du = \right\} 4[\ln u - u]_2^1 = 4[(\ln 1 - 1) - (\ln 2 - 2)] = 4(1-\ln 2)$ *Alternative method for part (d): Using an extra constant λ from their integration.* $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - [4\ln(1+\cos 0) - 4\cos 0 + \lambda]$ λ is usually -4, but can be a value of k that the candidate has found in part (d). **Note:** The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

44.

Question Number	Scheme		Marks
8. (a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\int \frac{1}{P(5-P)}\,dP = \int \frac{1}{15}\,dt$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ (+c)$		M1* A1ft
	$\{t=0, P=1 \Rightarrow\}\ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c\ \ \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \frac{5e^{\frac{1}{3}t}}{(4 + e^{\frac{1}{3}t})}\ \ \left\{\frac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \frac{5}{(1 + 4e^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4e^{-\frac{1}{3}t} > 1\ \Rightarrow\ P < 5$. So population cannot exceed 5000.		B1
			[1] **12**
(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question. **A1:** Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$. **A1:** $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\frac{1}{5P} + \frac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working. Candidate can use 'cover-up' rule to write down $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$, as so gain all three marks. Candidate cannot gain the marks for part (a) in part (b).		

8. (b)

B1: Separates variables as shown. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

M1*: **Both** $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants.

Or $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5 \pm P))$, where λ, μ, m and n are constants.

A1ft: Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$

with or without $+c$

dM1*: Use of $t = 0$ and $P = 1$ in an integrated equation containing c

dM1*: Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY.

dM1*: Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation.

dM1*: A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)**

A1: $P = \frac{5}{(1+4e^{-\frac{1}{3}t})}$ $\{$where $a = 5, b = 1, c = 4\}$.

Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5+20e^{-\frac{1}{3}t})}$

Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.

Note: $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft.

dM1* for making P the subject

Note there are three type of manipulations here which are considered acceptable to make *P* the subject.

(1) M1 for $\frac{P}{5-P} = e^{\frac{1}{3}t} \Rightarrow P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(1+e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1+e^{-\frac{1}{3}t})}$

(2) M1 for $\frac{P}{5-P} = e^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = e^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = e^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = e^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1+e^{\frac{1}{3}t})}$

(3) M1 for $P(5-P) = 4e^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4e^{\frac{1}{3}t} \Rightarrow \left(P - \frac{5}{2}\right)^2 - \frac{25}{4} = -4e^{\frac{1}{3}t}$ leading to $P = ...$

Note: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*.

Note: $(P) - (5-P) = e^{\frac{1}{3}t} \Rightarrow 2P - 5 = \frac{1}{3}t$ leading to $P = ...$ or equivalent is awarded this dM0*

(c)

B1: $1 + 4e^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000.

For $P = \frac{25}{(5+20e^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20e^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000.

B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b + ce^{-\frac{1}{3}t})}$.

Award B0 for: As $t \to \infty$, $e^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000,

unless the candidate also proves that $P = \frac{5}{(1+4e^{-\frac{1}{3}t})}$ oe. is an increasing function.

If unsure here, then send to review!

8.	*Alternative method for part (b)*
	B1M1*A1: as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ (+c)$
	Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$
	Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$
	Award 2nd M1 for $t=0,\ P=1 \Rightarrow \frac{1}{5-1} = Ae^{0} \ \left\{\Rightarrow \ A = \frac{1}{4}\right\}$
	$\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$
	then award the final M1A1 in the same way.

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45.

Question Number	Scheme	Marks
4.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ \ldots]$	B1
	$\approx \ \ldots\ \left[0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,dx$	B1
	$\int x^3 \ln(x^2+2)\,dx = \int x^2 \ln(x^2+2)\,x\,dx = \int (u-2)(\ln u)\tfrac{1}{2}\,du$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,du$ ✱ cso	A1 **(4)**

	(d) $\int (u-2)\ln u \, du = \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u^2}{2} - 2u\right)\frac{1}{u} du$	M1 A1	
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u}{2} - 2\right) du$		
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right) \quad (+C)$	M1 A1	
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right)\right]_2^4$		
	$= \frac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1	
	$= \frac{1}{2}(2\ln 2 + 1)$ $\quad \ln 2 + \frac{1}{2}$	A1	**(6)**
			[15]

46.

Question Number	Scheme	Marks
8.	(a) $\int (4y+3)^{-\frac{1}{2}}\,dx = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,dy = \int \frac{1}{x^2}\,dx$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,dy = \int x^{-2}\,dx$	
	$\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \frac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\frac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$	M1
	$y = \frac{1}{4}\left(2 - \frac{2}{x}\right)^2 - \frac{3}{4}$ or equivalent	A1 **(6)**
		[8]

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47.

Question Number	Scheme	Marks
1.	$\displaystyle\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$	M1 A1 A1
	$= \quad \dots \quad + \dfrac{\sin 2x}{4}$	M1
	$\left[\ \dots\ \right]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$	M1 A1
		[6]

48.

Question Number	Scheme	Marks
3.		
(a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $5 = 5A \Rightarrow A = 1$	**M1 A1**
	$x \to -\dfrac{2}{3}$ $5 = -\dfrac{5}{3}B \Rightarrow B = -3$	**A1** (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int\left(\frac{1}{x-1} - \frac{3}{3x+2}\right)dx$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	**M1 A1ft A1ft**
		(3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int\left(\frac{1}{y}\right)dy$	**M1**
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	**M1 A1**
	$y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	**M1 dep**
	Using $(2, 8)$ $8 = \dfrac{K}{8}$ depends on first two Ms in (c)	**M1 dep**
	$y = \dfrac{64(x-1)}{3x+2}$	**A1** (6)
		[12]

49.

Question Number	Scheme	Marks
7. (a)	$x=3 \Rightarrow y=0.1847$ awrt	B1
	$x=5 \Rightarrow y=0.1667$ awrt or $\frac{1}{6}$	B1
		(2)
(b)	$I \approx \frac{1}{2}\left[0.2+0.1667+2(0.1847+0.1745)\right]$	B1 M1 A1ft
	≈ 0.543 0.542 or 0.543	A1 (4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}u}=2(u-4)$	B1
	$\int \frac{1}{4+\sqrt{(x-1)}}\mathrm{d}x=\int \frac{1}{u}\times 2(u-4)\,\mathrm{d}u$	M1
	$=\int\left(2-\frac{8}{u}\right)\mathrm{d}u$	A1
	$=2u-8\ln u$	M1 A1
	$x=2 \Rightarrow u=5,\ x=5 \Rightarrow u=6$	B1
	$\left[2u-8\ln u\right]_5^6=(12-8\ln 6)-(10-8\ln 5)$	M1
	$=2+8\ln\left(\frac{5}{6}\right)$	A1
		(8) [14]

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50.

Question Number	Scheme	Marks
1.	(a) $y\left(\frac{\pi}{6}\right)\approx 1.2247,\ y\left(\frac{\pi}{4}\right)=1.1180$ accept awrt 4 d.p.	B1 B1 **(2)**
	(b)(i) $I \approx \left(\frac{\pi}{12}\right)(1.3229+2\times 1.2247+1)$ B1 for $\frac{\pi}{12}$	B1 M1
	≈ 1.249 cao	A1
	(ii) $I \approx \left(\frac{\pi}{24}\right)(1.3229+2\times(1.2973+1.2247+1.1180)+1)$ B1 for $\frac{\pi}{24}$	B1 M1
	≈ 1.257 cao	A1 **(6)** **[8]**

51.

Question Number	Scheme		Marks	
2.	$\frac{du}{dx} = -\sin x$		B1	
	$\int \sin x\, e^{\cos x+1}\,dx = -\int e^u\,du$		M1 A1	
	$= -e^u$	ft sign error	A1ft	
	$= -e^{\cos x+1}$			
	$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$	or equivalent with u	M1	
	$= e(e-1)$ ✱	cso	A1	**(6)**
				[6]

52.

Question Number	Scheme		Marks	
6.	(a) $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ ✱	cso	A1	**(3)**
	(b) $\int \theta\cos 2\theta\,d\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta\,d\theta$		M1 A1	
	$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta f(\theta)\,d\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\ \dots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16} - \frac{7}{4}$		A1	**(7)**
				[10]

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53.

Question Number	Scheme		Marks	
Q2	(a) 1.386, 2.291	awrt 1.386, 2.291	B1 B1	(2)
	(b) $A \approx \frac{1}{2} \times 0.5(\dots)$		B1	
	$= \dots \left(0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545\right)$		M1	
	$= 0.25\left(0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545\right)$	ft their (a)	A1ft	
	$= 0.25 \times 29.477 \dots \approx 7.37$	cao	A1	(4)
	(c)(i) $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$		M1 A1	
	$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$			
	$= \frac{x^2}{2} \ln x - \frac{x^2}{4} \; (+C)$		M1 A1	
	(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^4 = (8 \ln 4 - 4) - \left(-\frac{1}{4}\right)$		M1	
	$= 8 \ln 4 - \frac{15}{4}$			
	$= 8(2 \ln 2) - \frac{15}{4}$	$\ln 4 = 2 \ln 2$ seen or implied	M1	
	$= \frac{1}{4}(64 \ln 2 - 15)$	$a = 64, b = -15$	A1	(7)
				[13]

54.

Question Number	Scheme		Marks	
Q5	(a) $\int \frac{9x+6}{x}\,\mathrm{d}x = \int \left(9+\frac{6}{x}\right)\mathrm{d}x$		M1	
	$= 9x + 6\ln x \ (+C)$		A1	(2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$	Integral signs not necessary	B1	
	$\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$			
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \ (+C)$	$\pm k y^{\frac{2}{3}} =$ their (a)	M1	
	$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \ (+C)$	ft their (a)	A1ft	
	$y = 8,\ x = 1$			
	$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

55.

Question Number	Scheme		Marks	
Q8	(a) $\frac{dx}{du} = -2\sin u$		B1	
	$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,du$		M1	
	$= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,du$	Use of $1-\cos^2 u = \sin^2 u$	M1	
	$= -\frac{1}{4}\int \frac{1}{\cos^2 u}\,du$	$\pm k\int \frac{1}{\cos^2 u}\,du$	M1	
	$= -\frac{1}{4}\tan u \ (+C)$	$\pm k\tan u$	M1	
	$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \frac{\pi}{4}$			
	$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \frac{\pi}{3}$		M1	
	$\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$			
	$= -\frac{1}{4}\left(1-\sqrt{3}\right) \quad \left(= \frac{\sqrt{3}-1}{4}\right)$		A1	(7)
	(b) $V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x(4-x^2)^{\frac{1}{4}}}\right)^2 dx$		M1	
	$= 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$	$16\pi\times$ integral in (a)	M1	
	$= 16\pi\left(\frac{\sqrt{3}-1}{4}\right)$	$16\pi\times$ their answer to part (a)	A1ft	(3)
				[10]